

# Plan of Research

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Recently, it is shown that double scaling limits of the multi-critical unitary matrix model toward their multi-critical point is related to the four-dimensional supersymmetric theories of Argyres-Douglas type. They provide good examples of “gauge theory/matrix model correspondence”. We have been studying the multi-critical unitary matrix models in the large- $N$  limit. Here  $N$  is the size of the unitary matrix. The 1-cut solutions to the saddle point equation are obtained. Using the solution, the free energy and the Wilson loops in the large- $N$  limit are explicitly determined.

First, we will study more general  $n$ -cut solutions to the saddle point equation. Then, we try to consider finite  $N$  corrections and instanton corrections.

The simplest case of the multi-critical unitary matrix model is the Gross-Witten-Wadia (GWW) model with a logarithmic potential. It is known that the partition function of the GWW model is regarded as the tau function of the Painlevé III equation. It would be interesting to make clear to what integrable systems the general multi-critical models are related.

We also consider whether it is possible to  $q$ -deform these matrix models. The  $q$ -deformed models are expected to be related to five-dimensional gauge theories and six-dimensional theories, therefore analyzing their properties will be an interesting research topic. These theories are also expected to be related to the  $q$ -deformed two-dimensional field theories, whose symmetry algebras form the  $q$ -Virasoro/ $q$ -W algebras.

Previously, we have studied the root of unity limit of  $q$  in two dimensional models and showed that the parafermion algebra can be obtained from the  $q$ -Virasoro/ $q$ -W algebras. By studying similar limits in detail, we would like to clarify various properties of the “gauge theory/matrix model correspondence”. One of the research topics that we would like to try is how the limit is extended in the case of gauge theories on ALE spaces other than  $A$ -type gauge theories and the quiver gauge theories. A “Yangian algebra” associated with the quiver gauge theories has been proposed. In the Schur-Weyl correspondence, the Yangian algebra is related to the Hecke algebra, so an extension in this direction will deepen our understanding of the “gauge theory/conformal field theory/matrix model correspondence”.