Research Plan (2022)

(Masa-Hiko Saito)

The following is my research plan for 2022–2026.

1. **monodromy preserving deformation and asymptotic expansion geometry and its applications**

   (a) We will introduce canonical coordinates by means of apparent singularities in the moduli spsace of parabolic connections and parabolic Higgs bundles and we will constuct the explicit construction of the universal family of connections. Then by using the explicit univesal families of connections we describe the differential equations of monodromy preserving deformations

   (b) We will understand and describe the geometric strucuture of the moduli space of the parabolic connections and the parabolic Higgs bundle bythe moduli space of the h-parabolic connection. Then we will investigate the geometry of asymptotic expansions, including WKB analysis on the moduli spaces.

   (c) Geometry of spectral curves of parabolic Higgs bundles and applications of monodromy preserving deformations to various geometries.

2. **The correspondence between the asymptotic expansion of solutions of integrable systems on modular spaces and mathematical physics theory.**

   (a) We will deepen the understanding of asymptotic expansions of solutions of equations of Painlevé-type and τ-functions by WKB analysis.

   (b) We will reconsider the topological asymptotics of quantum curves of Eynard and coworkers in our framework.

   (c) We will understand the mathematical and physical meaning of the quantities calculated by Eynard's topological asymptotic formula for quantum curves. Understand the mathematical and physical meaning of the quantities calculated by Eynard's quantum curve.

3. **The algebraic and differential geometric study of moduli spaces and the basic theory and symmetry of the Fourier-Mukai transform**

   (a) We will study the structure of moduli spaces of connections and vector bundles.

   (b) We would like to determine the cohomology of a candidate sheaves on the moduli spaces for the kernel of the Fourier-Mukai transform and would like to study the structure of the projections between moduli spaces.

   (c) We would like to develop a geometric framework for understanding the symmetryof moduli spaces.