Summary of the study <small>(Yosuke Saito)</small>

The author researches mathematics related to conformal field theory and quantum many-body systems. Conformal field theory is a 2-dimensional quantum field theory which possesses a symmetry of the Virasoro algebra, and the theory is applied to some materials, for example superstring theory as well as statistical mechanics. The author is especially interested in the elliptic Calogero-Moser system and the elliptic Ruijsenaars system. In general, a Calogero-Moser system is a quantum system described by a many-body Schrödinger equation. If the Hamiltonian of the Calogero-Moser system has a rational (resp. trigonometric or elliptic) potential function, the system called the rational (resp. trigonometric or elliptic) Calogero-Moser system. Thus there are three types of Calogero-Moser system. Ruijsenaars system is a q-deformation of Calogero-Moser system. Similarly to Calogero-Moser system, there are three types of Ruijsenaars system called rational, trigonometric, and elliptic. Free field realization is a method which enable us to realize the action of a Hamiltonian of a quantum system or an representation of an algebra by using algebras of bosons. It had been known around 1996 that the trigonometric Calogero-Moser system and the Virasoro algebra, as well as the trigonometric Ruijsenaars system and the q-Virasoro algebra are related by free field realization. In 2000s, a different relation between the trigonometric Ruijsenaars system and a certain quantum algebra was discovered. We call the Hamiltonian of the trigonometric Ruijsenaars system the trigonometric Ruijsenaars operator. It turned out that an representation of a quantum group called the Ding-Iohara-Miki algebra is obtained from the free field realization of the trigonometric Ruijsenaars operator. Representations of the Ding-Iohara-Miki algebra are applied in physics, for example, topological vertex which is used for calculation of partition functions of topological string theory are recovered from correlation functions of intertwining operators for the Ding-Iohara-Miki algebra.

From the above, there are several related topics on the trigonometric Calogero-Moser system and the trigonometric Ruijsenaars system, however, there are not many known fact on the elliptic Calogero-Moser system and the elliptic Ruijsenaars system because of these complexities. In 2013, the author constructed the free field realization of the Hamiltonian of the elliptic Ruijsenaars system, and proposed that an representation of an elliptic deformation of the Ding-Iohara-Miki algebra is obtained from the free field realization. However, many problems on the elliptic Ruijsenaars system still remain open. The author will describe more concrete topics on the elliptic Ruijsenaars system in "Plan of the study".