Research Results

I have been working on knot theory and 3-manifold theory, and my main results are as follows.

(1) Branched coverings. When I began to study knot theory as a graduate student, the Alexander polynomial and homology of branched coverings were essentially the only invariants of knots. So, my main concern was to get full understanding of these invariants. In my first paper, I gave a simple alternative proof to the Hosokawa-Kinoshita formula on the homology of finite cyclic coverings of links, and in the succeeding paper, I generalized the Murasugi formula on the Alexander polynomials of periodic links to a formula on periodic links. I eventually gave a formula which expresses the Betti numbers of finite abelian coverings of links in terms of the Alexander invariants, and applied it to obtain a generalization of E. Hironaka's polynomial periodicity of towers of abelian coverings of algebraic surfaces. This result has been used in the study of algebraic surfaces. I also gave the branched fibration theorem that relates Heegaard surfaces and fibered surfaces via double branched covering construction. Immediately after its publication, it was refined by R. Brooks and J. M. Montesinos, independently, and recently, it was also refined by a joint work by S. Hirose and E. Kin.

(2) Symmetry of knots. Inspired by the works described in (1), I was interested in symmetries of knots. By a joint work with Kouji Kodama, we tried to determine the symmetry groups of prime knots up to 10 crossings; in conjunction with the independent joint work by S. Henry and J. Weeks, the symmetry groups all such knots were completely determined. Since then, I studied strongly invertible knots, freely periodic knots, uniqueness of symmetries, realization problem of symmetry groups, etc. In a joint work with Luisa Paoluzzi, we succeeded in solving a problem concerning relation between free period and chirality, which I had proposed in 1986.

(3) Unknotting tunnels for knots. In relation with the works described in (2), I was interested in Heegaard splittings of 3-manifolds, in particular unknotting tunnels for knots. In a series of joint works with Kanji Morimoto and Yoshiyuki Yokota, we gave a complete classification of tunnel number 1 satellite knots and their unknotting tunnels, a construction of tunnel number 1 knots which do not admit $(1,1)$-decompositions, and a condition for Montesinos knots to have tunnel number 1. In a joint work with Elena Klimenko, motivated by the last result, we solved the problem to determine when a given pair of orientation-reversing isometries of the hyperbolic plane generates a discrete group. As a corollary, we determined the ranks of triangle groups, which in turn leads us to the complete classification of tunnel number 1 Montesinos knots. The success of this work gave me a courage to embark on the project described in (4).

(4) 2-bridge knots and punctured torus groups. There are intimate relation between the two simple objects, the 2-*bridge knots*, the simplest hyperbolic knots which play important role in knot theory, and the *punctured torus*, the simplest hyperbolic surface which admit the nontrivial deformation (Theichmüller) space. It has been my main research theme, for these 20 years, to reveal the beautiful and deep mathematical structures hidden in these two simple objects, and then to study, through understanding of these structures, the topology and geometry of general knots and 3-manifolds. The starting point of this project is the joint work with Jeffrey Weeks which presents topological ideal triangulations of hyperbolic 2-bridge link complements that are conjectural candidates of the (Epstein-Pener) canonical decompositions. Following Weeks' suggestions that Jorgensen's unfinished work on quasifuchsian punctured torus groups should hold a key to the proof of our conjecture, I launched to a joint project with Hirotaka Akiyoshi, Masaaki Wada, and Yasushi Yamashita to understand Jorgensen's work. As a result of long-term collaboration, we published a lecture note (Springer) that includes a complete description and proof of Jorgensen's theory on quasifuchsian punctured torus groups, and an outline of an extension of Jorgensen's theory to the outside of quasifuchsian spaces, together with its application to the proof of the conjecture. In relation with this project, we gave a generalization of Epstein-Penner decomposition for cusped hyperbolic manifolds of finite volume to the case of infinite volume (joint work with Akiyoshi), variations of McSahen's identity for quasifuchsian punctured surface groups and for punctured surface bundles over the circle (joint works with Akiyoshi and Hideki Miyachi), and a variation of McSahen's identity for 2-bridge links (a series of joint works with Donghi Lee). In a joint work with Ken'ichi Ohshika, we gave a partial positive answer to conjectures which naturally arose from the above joint works with Lee. In a joint work with Brian Bowditch, we studied the mapping class group action on the space of geodesic rays for a punctured surface; this is a first step towards generaization of the results for 2-bridge links to general hyperbolic links. In a joint work with Yuya Koda, we introduced the concept of a homotopy motion group, and gave a systematic study of homotopy motion groups of surfaces in closed orientable 3-manifolds.