Research Results

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My main results are as follows:

(1) Examples of nonsingular complete toric varieties that are not quasitoric manifolds

The orbit space $X/(S^1)^n$ of a nonsingular complete toric variety X of complex dimension n by the restricted action of the compact torus $(S^1)^n \subset (\mathbb{C}^*)^n$ is a manifold with corners such that all faces are contractible and any nonempty intersection of faces is connected. If X is projective or $n \leq 3$, then the orbit space $X/(S^1)^n$ is homeomorphic to a simple polytope as a manifold with corners. I constructed nonsingular complete toric varieties of complex dimension ≥ 4, whose orbit spaces by the action of the compact torus are not homeomorphic to simple polytopes. These provide the first known examples of nonsingular complete toric varieties that are not quasitoric manifolds.

(2) Toric Fano varieties associated to building sets

There is a construction of nonsingular projective toric varieties from building sets, which are formed by subsets of a finite set. Such varieties were first studied by De Concini–Procesi as smooth completions of hyperplane arrangement complements in a projective space and they are now called wonderful models. Building sets were originally defined as subspace arrangements with some suitable properties. The class of toric varieties associated to building sets includes projective spaces and toric varieties corresponding to graph associahedra of finite simple graphs. I gave a necessary and sufficient condition for the toric variety associated to a building set to be Fano or weak Fano in terms of the building set.

(3) Fano generalized Bott manifolds

A generalized Bott manifold is a nonsingular projective variety obtained as the total space of an iterated complex projective space bundles over a point, where each fibration is the projectivization of the direct sum of several line bundles. Any generalized Bott manifold has the structure of a toric variety, and it is determined by a collection of integers. I gave a necessary and sufficient condition for a generalized Bott manifold to be Fano or weak Fano. As a consequence I characterized Fano Bott manifolds.

(4) Toric Fano varieties with positive second Chern characters

(joint work with Yuji Sano and Hiroshi Sato) We constructed the first known examples of $\mathbb{Q}$-factorial terminal toric Fano varieties of Picard number two such that the sum of the squared torus-invariant prime divisors is positive. By a computer calculation, we verified that any nonsingular toric Fano variety of dimension at most eight with the positive second Chern character is isomorphic to the projective space.