(英訳文)

I major in complex geometry and geometric analysis. Especially, I am interested in the existence problem of canonical geometric structures in a given (co)homology class. A typical example is the uniformization theorem for Riemann surfaces, which says that every Riemann surface admits a metric with constant curvature, and is classified according to the sign of the curvature. In this vein, the existence problem of canonical geometric structures has a deep connection with the classification and moduli problem of manifolds. The question is whether there exist canonical geometric structures on more general, higher dimensional manifolds.

Around Yau–Tian–Donaldson conjecture

Yau–Tian–Donaldson (YTD) conjecture says that a polarized manifold (X, L) admits a constant scalar curvature Kähler metric if and only if it is K-stable (a stability notion in algebraic geometry). In general, dynamical systems/geometric flows are effective tools to construct canonical metrics. In [2,8,10,14,16], I introduced new dynamical systems/geometric flows on the space of Kähler metrics, and showed the convergence under some stability condition. Meanwhile, we can embed X into a projective space $\mathbb{CP}^{N_k}$ ($N_k := \dim H^0(X, L^{\otimes k})$) by using global holomorphic sections of $L^{\otimes k}$. From this point of view, one can ask the relation between the existence of canonical metrics and canonical embeddings $X \hookrightarrow \mathbb{CP}^{N_k}$ called **balanced embeddings** for k sufficiently large. In [5,9,10,11,13], I studied this for several kinds of canonical metrics/geometric flows by means of the asymptotic expansion of Bergman kernels. As for a study related to Mumford's Geometric Invariant Theory (GIT), in [12], I exploit some formulas to compute the GIT weights associated to canonical metrics and its quantizations. Also, in [2,6], we studied the blowup behavior of geometric flows when the manifold admits no canonical metrics. As a consequence, we showed that the flow forms singularities along a subscheme, whose associated test configuration optimizes a non-Archimedean energy functional among all test configurations.

Around Thomas–Yau conjecture

Let X be a Calabi–Yau manifold and $\Sigma \subset X$ a Lagrangian. **Thomas–Yau (TY) conjecture** asserts that the hamiltonian isotopy class $[\Sigma]$ can be represented by a special Lagrangian if and only if $[\Sigma]$ is "stable". Although it is expected that the Bridgeland stability condition for Fukaya category is a candidate for such a stability condition, we have many unknowns about it and are even hard to formulate it in general. In [7], we studied the behavior of the mean curvature flow for real surfaces by means of hyperkähler structures when the ambient space X has complex dimension 2. Consequently, we proved that any special Lagrangian is linearly stable (i.e. stable for its small deformations) along the mean curvature flow.

Meanwhile, in the context of Strominger–Yau–Zaslow mirror symmetry, it is known that special Lagrangian equation corresponds to the **deformed Hermitian–Yang–Mills (dHYM) equation** for fiber metrics on a holomorphic line bundle $L \to X$. This complex geometric view point leads us to obtain several consequences. First, in [3], I studied the limiting behavior of the line bundle mean curvature flow ("mirror" of the Lagrangian mean curvature flow) when X does not admits any dHYM metrics, and showed that the flow blows up along curves of negative self-intersection as time goes to infinity. This result suggests a deep connection between the dHYM equation and algebro-geometric stabilities. Next, in [4], I exploit a new geometric flow and showed that for any initial almost calibrated data, the flow exists for all time and converges to the dHYM metric if it exists. Moreover, in [1], we give a necessary and sufficient condition for the existence in terms of algebraic geometry, as an analogue of the Nakai–Moishezon ampleness criterion for holomorphic line bundles. In [15], I introduced a vector bundle version of the J-equation originally studied by S. K. Donaldson and X. Chen in the line bundle case, and explore basic properties as well as examples of them. As an application, I constructed solutions to a vector bundle version of the dHYM equation as a small deformation of a solution to the J-equation on holomorphic vector bundles of arbitrary rank.