Research results

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On the braid index of Kanenobu knots Every knot is presented as a closed braid. The braid index of a knot is the minimum number of strings of a braid needed for the knot to be presented as a closed braid. A lower bound of the braid index of a knot is given by applying the Khovanov-Rozansky homology. Since Kanenobu knots $k(n)$ $(n = 0, 1, 2, \dots)$ have the same Khovanov-Rozansky homology, it is not easy to determine the braid index $\beta(k(n))$ of $k(n)$. We give a sharper lower bound of $\beta(k(n))$ by applying the $\Gamma_{2/q}$-polynomial.

The $\Gamma_{p/q}$-polynomial for mutant knots It is known that many knot invariants are invariant under mutation, for example, the $\nabla_{p/q}$, $V_{p/q}$, P, F, $P_{2/q}$, $F_{2/q}$-polynomials are invariant under mutation. On the other hand, the $P_{3/q}$-polynomial distinguishes a mutant knot pair. We show that the $\Gamma_{3/q}$-polynomial which is contained in the $P_{3/q}$-polynomial is invariant under mutation.

On the arc index of Kanenobu knots Every knot has an arc presentation. The arc index of a knot is the minimum number of pages needed for the knot to be presented as an arc presentation. The Morton-Beltrami inequality gives a lower bound of the arc index of a knot by applying the a-span of the Kauffman polynomial. Since Kanenobu knots $k(n)$ $(n = 0, 1, 2, \dots)$ have the same a-span of the Kauffman polynomials, it is not easy to determine the arc index $\alpha(k(n))$ of $k(n)$. We construct "canonical cabling algorithm" which gives sharper upper bounds of the arc index of cable knots and give a sharper lower bound of $\alpha(k(n))$ by applying "canonical cabling algorithm" and the $\Gamma_{2/q}$-polynomial. (This is a joint work with Hwa Jeong Lee.)

A characterization of the Γ-polynomials of knots with clasp number at most two Every knot bounds a singular disk with only clasp singularities, which is called a clasp disk. The clasp number of a knot is the minimum number of clasp singularities among all clasp disks of the knot. It is known that the Alexander-Conway polynomials of knots with clasp number at most two are characterized. We characterize the Γ-polynomials of knots with clasp number at most two.

On knots with the trivial $\Gamma_{2/1}$-polynomial For the trivial knot $\bigcirc$, $\nabla_{p/1}(\bigcirc) = V_{p/1}(\bigcirc) = \Gamma_{p/1}(\bigcirc) = Q_{p/1}(\bigcirc) = P_{p/1}(\bigcirc) = F_{p/1}(\bigcirc) = 1$ for any integer $p(\geq 2)$. It is known that there exists a non-trivial knot K such that $\nabla_{p/1}(K) = 1$ for any integer $p(\geq 2)$. We consider whether there exist an integer $p(\geq 2)$ and a non-trivial knot K such that $I_{p/1}(K) = 1$ for $I = V, \Gamma, Q, P, F$. In particular, we show that there exist infinitely many knots with the trivial $\Gamma_{2/1}$-polynomial.

The $\Gamma_{2/1}$-polynomial of knots up to ten crossings Since it is known that the Γ-polynomial is computable in polynomial time, the $\Gamma_{p/q}$-polynomial is also computable in polynomial time. We show that the $\Gamma_{2/1}$-polynomial completely classifies the unoriented knots with up to ten crossings including the chirality information.

The self-smoothing number of knots and links We call smoothing a self-crossing point of an oriented link diagram self-smoothing. By self-smoothing repeatedly, we obtain an oriented link diagram without self-crossing points. We show that every knot has an oriented diagram which becomes a two-component oriented link diagram without self-crossing points by a single self-smoothing.

Classification of Abe-Tange's ribbon knots Abe and Tange constructed a sequence of slice disks with the same exterior. Moreover, they showed that these slice disks are ribbon disks. We call the boundaries of the ribbon disks Abe-Tange's ribbon knots. We classify Abe-Tange's ribbon knots completely by using the Γ-polynomial.

Vassiliev knot invariants derived from the $\Gamma_{p/q}$-polynomials We give some results on Vassiliev knot invariants derived from the $\Gamma_{p/q}$-polynomials. In particular, we show that all Vassiliev knot invariants of order ≤ 4 are determined by the $\Gamma_{p/q}$-polynomials.

$2n$-moves and the Γ-polynomial for knots A $2n$-move is a local change for knots and links which changes $2n$ half twists to 0 half twists or vice versa for a natural number n. In 1979, Yasutaka Nakanishi conjectured that the 4-move is an unknotting operation. This is still an open problem. In particular, we show that the $4k$-move is not an unknotting operation for any integer $k(\geq 2)$ by using the Γ-polynomial, and if $\Gamma(K; -1) = 9 \pmod{16}$ then the knot K cannot be deformed into the unknot by a single 4-move. Moreover, we consider the 4-move distance of knots, which is the minimal number of 4-moves needed to deform one into the other. In particular, the 4-move unknotting number of a knot is the 4-move distance to the unknot. We give a table of the 4-move unknotting number of knots with up to 9 crossings. (This is a joint work with Taizo Kanenobu.)