About my past research

My main interest has been on the duality.

In the student time (in the master and doctor course), I started my study on representations from the functional analysis point of view. I studied some general theory on representations and also construction of concrete representations. Among them, I gave a much fairly simplified proof of (weak) Tatsuuma duality theorem on the general locally compact groups[1], some duality theorem on $L^\infty(G)$ for locally compact group G [2], and also constructed representations of the universal covering group of symplectic groups, which are motivated by the study of Kubota on the higher reciprocity law.

In my thesis, I turned to more algebraic theme, on the conjugacy of nilpotent elements of the classical Lie algebras, in which not only the representation theory of symmetric groups but properties of cyclotomic polynomials are fully utilized. In other words, number theoretic view points are adopted systematically.

These were, sort of sporadic results in my carrier. Much more systematic research began when I went to Yale University in 1989. The joint work with professor Roger Howe has been recognized as a break-through point of view on Capelli identities. One of his major work has been known as the name "theory of dual pair", and the Capelli identities are, in one sense, the incarnation of the duality, expressed in terms of invariant differential operators.

Further work on the Capelli identities are progressed in various directions, e.g, to the quantum groups and also treating different types of central elements and/or different Lie algebras.

Also, sort of simplified techniques treating this kind of identities are systematically elaborated by myself, my students, and colleagues, so that new and unified points of view have been developed.

Some expositions are given in the papers [13] and [17], and also in many Japanese expositions by myself.

Through those studies and also from the different motivations, I revealed that some very old papers have important meanings even in the modern mathematics. It is very much interesting to do so, as natural sciences other than mathematics hardly reach such results.