RESEARCH RESULTS

HIDEYA WATANABE

My research interests are representation theory of Lie algebras and applications to related mathematics and mathematical physics. Especially, recently I am interested in the $\imath$quantum groups, which are algebraic systems appearing in the theory of quantum symmetric pairs. A quantum symmetric pair is a quantization of certain pair of complex Lie algebras. To be more specific, it is a pair of a Drinfeld-Jimbo quantum group and its certain coideal subalgebra. Such a subalgebra is referred to as an $\imath$quantum group. Based on an idea "The $\imath$quantum groups are generalizations of the quantum groups"($\imath$program), I aim to generalize important results in the theory of quantum groups to the $\imath$quantum groups setting.

(1) **Representation theory of the $\imath$quantum group of type AIII.**

Bao-Wang proved that the $\imath$quantum group of type AIII and the Hecke algebra of type B(=C) are in Schur duality. In joint works with them, I extended this result to the unequal parameter case. In particular, this extension covers the asymptotic case. Applying the representation theory of the Hecke algebra of type B with asymptotic parameter via Schur duality, I described the structures of finite-dimensional irreducible modules over the $\imath$quantum group of type AIII in detail.

Also, by extending the theory of $\imath$canonical bases due to Bao-Wang, they and I determined the irreducible characters of the ortho-symplectic super Lie algebra completely.

(2) **Classical weight modules.**

I defined the notion of classical weight modules in representation theory of the $\imath$quantum groups by generalizing the notion of weight modules in representation theory of the quantum groups. I showed that the classical weight modules form a good class in finite-dimensional representation theory of the $\imath$quantum groups. I classified the finite-dimensional classical weight module for the $\imath$quantum groups of type AI, AII, and AIII.

(3) **Representation theory of the $\imath$quantum groups and combinatorial structures.**

I introduced the notion of *$\imath$crystals* by abstracting combinatorial structures appearing in representations of certain quasi-split $\imath$quantum groups. It is a generalization of the notion of crystals appearing in the theory of quantum groups. It turned out that $\imath$crystal plays an essential role in the construction of the crystal basis of modified $\imath$quantum groups.

Date: March 25, 2022.