4 Research statement

I work in algebraic geometry in positive characteristic from a point of view of arithmetic geometry. My research results are divided into two parts.

4.1 Research on quasi-F-splitting

My greatest research work is to introduce the notion of *quasi-F-splitting*.

This is a notion for schemes in positive characteristic and generalizes the notion of *F-split*. Mehta-Ramanathan introduced the notion of F-split in 1980's and applied it to the study of cohomology of line bundles of Schubert varieties. A similar notion was studied by Hochster from the late 1960's in the tight closure theory, which is a part of commutative algebra. Nowadays, this grows into the theory of F-singularities and has vast applications to birational geometry.

Therefore, F-split theory has been developed in commutative algebra, geometric representation theory and algebraic geometry in positive characteristic. I started a study of F-splitting from the view point of arithmetic geometry. A motivating theme is a study of Calabi-Yau varieties.

M. Artin and B. Mazur defined the invariant of Artin-Mazur height, which take values in positive integers or infinity, for Calabi-Yau varieties in positive characteristic. This generalizes the difference between ordinarity and supersingularity of elliptic curves in positive characteristic and is defined using the theory of formal groups. This invariant captures the information of a part of crystalline cohomology and hence has an arithmetic flavor. According to previous research on $K3$ surfaces or three dimensional Calabi-Yau varieties, geometric properties (e.g., unirationality or singularity of deformation space) of varieties depend critically on whether the Artin-Mazur height is finite or not.

It is known that F-splitting is equivalent to Artin-Mazur height being one for Calabi-Yau varieties. I focused on this fact and started a research to generalize this bridge between splitting and arithmetic to capture Calabi-Yau varieties of finite Artin-Mazur height. I got the following results.

1. I introduced an invariant of *quasi-F-split height* for schemes in positive characteristic,
2. I proved that quasi-F-split height being one is equivalent to F-splitting, and
3. the quasi-F-split height is equal to the Artin-Mazur height for Calabi-Yau varieties.

Furthermore, I defined that a scheme is *quasi-F-split* if its quasi-F-split height is finite.

Here I will not state the precise definition, it uses the Witt ring of structure sheaf. Note that it is possible to define quasi-F-splitness in terms of coherent sheaves of the structure sheaf. Using this point effectively, I proved the following: If X is quasi-F-split, then

1. the scheme X can be lifted to modulo p^2 (here p denotes the characteristic of the ground field of X)
2. the projective variety X satisfies the Kodaira vanishing theorem.

Note that these two properties are not satisfied in general. Furthermore, I did a study on quasi-F-splitting of Enriques surfaces and abelian varieteis ([3]). Also, in a joint work with Yukiyoshi Nakkajima (Tokyo Denki University), we studied quasi-F-splitting for log schemes([4]).

In the previously stated researches, I mainly focused on “global” varieties. Then I started a study on quasi-F-split for singularities, especially two dimensional rational double points (RDP's). It is known that when the characteristic p of the ground field is greater than 5, all RDP's are F-split but there are non-F-split RDP's when p is less than or equal to 5. Recently, I proved that all RDP's are quasi-F-split, including the low characteristic case.

4.2 Research on the moduli of supersingular abelian varities and its mass formula

This is a jont work with Chia-Fu Yu and Valentijn Karemaker ([5]).

This research is a generalization of the Deuring's mass formula, which counts the number of supersingular elliptic curves with weight, to higher dimensional abelian varieties. Originally, this was related to computing the class number of an algebraic group and hence is a highly arithmetic problem. Our research uses the theory of the moduli space of supersingular abelian varieties by Li-Oort and has more geometric flavor. There are several formulations of this problem, we focused on the one used by Chia-Fu Yu and Jeng-Daw Yu when they studied the abelian surface case. We computed a mass formula for supersingular abelian threefolds and solves the Oort's conjecture on automorphism groups of principally polarized abelian varieties.