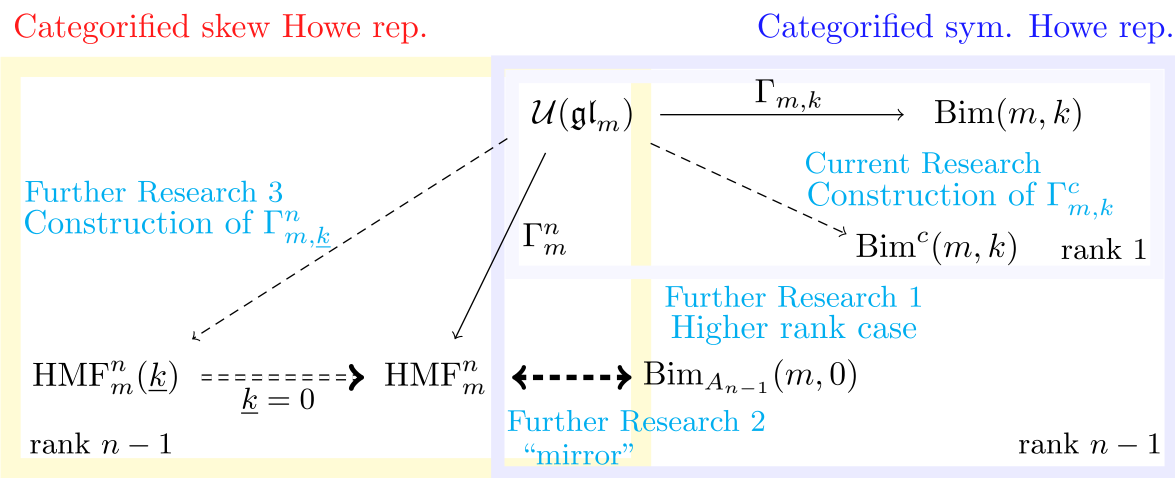

Solid arrows in the figure were results obtained in previous work. Dashed arrows indicate future areas of study.

Current Research: Currently, I have been constructing a generalization of the deformed Webster algebra of type A_1.

1. To construct a deformed Webster algebra $W^{\mathfrak{g}}$
2. To introduce a p-DG structure on the deformed Webster algebra $W^{\mathfrak{g}}$.

Further Research 1: On the symmetric product $S^k(\mathbb{C}^n \otimes \mathbb{C}^m)$, we have a left $U_q(\mathfrak{sl}_n)$ action and a right $U_q(\mathfrak{gl}_m)$ action such that these actions commute. So we have a representation

$$\gamma_m^{\mathfrak{sl}_n} : U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha = k,\, \sum_{\alpha=1}^m j_\alpha = k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(S^{i_1} \otimes \cdots \otimes S^{i_m}, S^{j_1} \otimes \cdots \otimes S^{j_m}).$$

It is expected that there exist a deformed Webster algebra $W^{A_{n-1}}(\mathbf{s}, \underline{k})$ of type A_{n-1} and a functor from $\mathcal{U}(\mathfrak{gl}_m)$ to the bimodule category $\mathrm{Bim}_{A_{n-1}}(m, \underline{k})$. The challenge is to find "good" bimodules in the category $\mathrm{Bim}_{A_{n-1}}(m, \underline{k})$, construct concrete bimodule morphisms and then define the functor. Furthermore, it is expected that its cyclotomic quotient $W^c(\mathbf{s}, k)$ is a cellular algebra.

Further Research 3: In the skew case, it is expected that there exists a category of deformed matrix factorizations $\mathrm{HMF}^n_m(\underline{k})$ with a parameter $\underline{k} \in \mathbf{Z}^{n-1}_{\geq 0}$ which is derived from the same concept as k in the algebra $W(\mathbf{s}, k)$. The category $\mathrm{HMF}^n_m(\underline{k})$ in the case $\underline{k} = 0$ is the category HMF^n_m. First, I'll consider the case of A_1.

Further Research 4: The p-DG structure is a generalization of DG structure whose derivation ∂ has the condition $\partial^p = 0$. Using this structure, we try to define homological three-manifold invariants.