Research Result

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• tensor model

The tensor model can be regarded as a generalization of rectangular matrix model to higher rank. Recently, the tensor model is receiving a lot of attention because of its relation to the low dimensional AdS/CFT and quantum gravity. However, unlike ordinary matrix models, the gauge-invariant operators in the tensor model have nontrivial structures, which make it difficult to use ordinary methods such as the Virasoro constraints.

In order to resolve the enumeration problem of the operators, I focused on the so-called cut operation. The cut operation corresponds to the variation of the measure of the partition function under change of variables. This cut operation can be extended to a "generalized cut" that includes higher-order contributions of the variation of the integral measure. I found that the cut operation can be used to generate a variety of operators. I also arrived at a conjecture about the selection of the appropriate variation function to generate all operators. In the case of the rank 3 model, it was confirmed that the conjecture is correct at least within the region examined.

On the other hand, I demonstrated the following Op/FD correspondence between the tensor models of different ranks.

$$\text{Operator (rank } r) \quad \Longleftrightarrow \quad \text{Feynman Diagram (rank } r-1)$$

Each operator in the tensor model is, therefore, labeled with the Feynman diagram. In particular, in the case of rank 3, it is extended to an Op/FD/dessin correspondence including a one-to-one correspondence with graphs called dessins. Here the dessin is a graph consisting of vertices of two colors and edges connecting them embedded on a two-dimensional surface. I succeeded in building a concrete relationship. By using this correspondence, all operators up to level 5 in the rank 3 tensor model were classified according to properties of FD and dessin, for example, the number of vertices. In addition, I have established the interpretation of the cut & join operations as diagrammatic manipulations by expressing them in the language of dessin.

• 2d/4d(5d) correspondence

The 2d/4d correspondence states the equivalence between the conformal block in 2d CFT and the instanton partition function in 4d supersymmetric gauge theory. The correspondence between the q-W_n conformal block and the 5d instanton partition function was also proposed. These have two parameters q and t. By taking the limit $q, t \to 1$, this correspondence reduces to the 2d-4d one. I have studied the root of unity limit in q and t. The generators of the q-Virasoro (q-W_2) algebra can be described by a q-boson field. In the $q, t \to -1$ limit, the generators of the superconformal algebra appear. Similarly, the free boson and free fermion which describe this algebra can be obtained from the q-boson in this limit. On the 5d side, I obtained the 5d instanton partition function in the root of unity limit which is the same as that used on the 2d side. I have confirmed that the results are equal to the 4d ALE instanton partition function at the lower level at least. The 2d/4d correspondence can be understood through the limiting procedure in the 2d/5d correspondence.

In the general r-th root of unity limit of q-W_n algebra, the $\mathbf{Z}_r$-parafermions appear. The obtained theory is the coset CFT which has $\frac{\widehat{sl}(n)_r \oplus \widehat{sl}(n)_p}{\widehat{sl}(n)_{r+p}}$ symmetry. In fact, the central charge of the energy-momentum tensor is exactly reproduced. The parameter p is related with the omega-background of the corresponding gauge theory and its relation was also clarified.

I considered also another root of unity limit ($q \to 1,\ t \to -1$). There exists the correspondence between a modified affine $sl(2)_k$ current block and the instanton partition function in the presence of a surface operator. The representation of affine $sl(2)_k$ algebra can be realized in terms of free fields which are obtained from the q-boson at the root of unity limit as mentioned above. I presented explicitly the free field representation of affine $sl(2)_k$ algebra and derived the integral representation of the modified current blocks.