AHA23051 後呂悠惟

C.1.2

(1)

$$\text{最大値を}(x,y)=(x_1,y_1)\text{で持つと仮定する。このとき、}$$
$$f\left(2x_1+\frac{c}{2a},y_1\right)-f(x_1,y_1)=3a\left(x_1+\frac{c}{2a}\right)^2\geq 0\text{より、}$$
$$f\left(2x_1+\frac{c}{2a},y_1\right)\geq f(x_1,y_1)\quad\text{もし}f\left(2x_1+\frac{c}{2a},y_1\right)>f(x_1,y_1)\text{であれば}$$
$$\text{仮定に反するため}f\left(2x_1+\frac{c}{2a},y_1\right)=f(x_1,y_1)\quad\text{等号成立条件より}$$
$$x_1+\frac{c}{2a}=0\quad\text{このとき、}f(x_1+1,y_1)-f(x_1,y_1)=a>0\text{より}$$
$$f(x_1+1,y_1)>f(x_1,y_1)\quad\text{これは仮定に反する。}$$

(2)

$$\text{最小値を}(x,y)=(x_1,y_1)\text{で持つと仮定する。このとき、}$$
$$f\left(x_1,2y_1+\frac{d}{2b}\right)-f(x_1,y_1)=3b\left(y_1+\frac{d}{2b}\right)^2\leq 0\text{より、}$$
$$f\left(x_1,2y_1+\frac{d}{2b}\right)\leq f(x_1,y_1)\quad\text{もし}f\left(x_1,2y_1+\frac{d}{2b}\right)<f(x_1,y_1)\text{であれば}$$
$$\text{仮定に反するため}f\left(x_1,2y_1+\frac{d}{2b}\right)=f(x_1,y_1)\quad\text{等号成立条件より}$$
$$y_1+\frac{d}{2b}=0\quad\text{このとき、}f(x_1,y_1+1)-f(x_1,y_1)=b<0\text{より}$$
$$f(x_1,y_1+1)<f(x_1,y_1)\text{これは仮定に反する。}$$

(3)

$$z=f(x,y)=ax^2+by^2+cx+dy=a\left(x+\frac{c}{2a}\right)^2+b\left(y+\frac{d}{2b}\right)^2-\frac{c^2}{4a}-\frac{d^2}{4b}$$
$$\text{よって}\quad a\left(x+\frac{c}{2a}\right)^2+b\left(y+\frac{d}{2b}\right)^2=z+\frac{c^2}{4a}+\frac{d^2}{4b}\quad z\neq-\frac{c^2}{4a}-\frac{d^2}{4b}\text{のとき}$$
$$\frac{\left(x+\frac{c}{2a}\right)^2}{\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{a}}-\frac{\left(y+\frac{d}{2b}\right)^2}{\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{b}}=1$$

(i) $z < -\frac{c^2}{4a} - \frac{d^2}{4b}$のとき、

$$\frac{\left(x+\frac{c}{2a}\right)^2}{\left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{a}\right|} - \frac{\left(y+\frac{d}{2b}\right)^2}{\left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{b}\right|} = -1$$

$$\therefore \text{双曲線}$$

$$\text{漸近線}: y = \pm\sqrt{-\frac{a}{b}}\left(x+\frac{c}{2a}\right) - \frac{d}{2b}$$

(ii)$z = -\frac{c^2}{4a} - \frac{d^2}{4b}$のとき、

$$a\left(x+\frac{c}{2a}\right)^2 + b\left(y+\frac{d}{2b}\right)^2 = 0 \quad (\sqrt{a})^2\left(x+\frac{c}{2a}\right)^2 - (\sqrt{-b})^2\left(y+\frac{d}{2b}\right)^2 = 0$$

$$\left(\sqrt{a}x + \frac{c}{2\sqrt{a}} + \sqrt{-b}\left(y+\frac{d}{2b}\right)\right)\left(\sqrt{a}x + \frac{c}{2\sqrt{a}} - \sqrt{-b}\left(y+\frac{d}{2b}\right)\right) = 0$$

$$y = -\sqrt{-\frac{a}{b}}x - \frac{c}{2\sqrt{-ab}} - \frac{d}{2b}, y = \sqrt{-\frac{a}{b}}x + \frac{c}{2\sqrt{-ab}} - \frac{d}{2b}$$

$$\therefore 2\text{本の直線}$$

(iii)$z > -\frac{c^2}{4a} - \frac{d^2}{4b}$のとき、

$$\frac{\left(x+\frac{c}{2a}\right)^2}{\left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{a}\right|} - \frac{\left(y+\frac{d}{2b}\right)^2}{\left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{b}\right|} = 1$$

$$\therefore \text{双曲線}$$

$$\text{漸近線}: y = \pm\sqrt{-\frac{a}{b}}\left(x+\frac{c}{2a}\right) - \frac{d}{2b}$$

$e = \left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{a}\right| + \left|\frac{z+\frac{c^2}{4a}+\frac{d^2}{4b}}{b}\right|$とすると、

(i)の焦点は$(0, \pm e)$, (ii)の焦点は$(\pm e, 0)$