AHA23308
田村 一路

C.2.3

(1)

$$\frac{\partial z}{\partial \mathbb{v}} = \lim_{t \to \infty} \frac{z\big((x,y)+t(p,q)\big) - z(x,y)}{t}$$

$$= \lim_{t \to 0} \frac{3(x+tp) - 4(y-tq) - 3x + 4y}{t}$$

$$= \lim_{t \to 0} \frac{3tp - 4tq}{t}$$

$$= \lim_{t \to 0} (3p - 4q)$$

$$= 3p - 4q$$

(2)

$p^2 + q^2 = 1$だから、

$$\exists \theta \in [0, 2\pi)\ s.t.\ p = \cos\theta, q = \sin\theta$$

よって、$3\cos\theta - 4\sin\theta$が最大値となる$\theta$を求めればいい

$$3\cos\theta - 4\sin\theta$$

$$= 5\cos(\theta + \alpha)$$

$$\text{ただし}\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}, \alpha \in (0, \frac{\pi}{2})$$

よって、$\theta = 2\pi - \alpha$のとき$3\cos\theta - 4\sin\theta$は最大

$$p = \cos(2\pi - \alpha) = \cos\alpha = \frac{3}{5}$$

$$q = \sin(2\pi - \alpha) = -\sin\alpha = -\frac{4}{5}$$

よって、$\mathbb{v} = \left(\frac{3}{5}, -\frac{4}{5}\right), \frac{\partial z}{\partial \mathbb{v}} = 3 \cdot \frac{3}{5} - 4 \cdot \left(-\frac{4}{5}\right) = 5$

(3)

$3p - 4q = 0$のとき、

$$p = \frac{4}{3}q$$

$p^2 + q^2 = 1$に代入して解くと、

$$p = \pm\frac{4}{5}, q = \pm\frac{3}{5}\text{(複号同順)}$$

よって、$\mathbb{v} = \pm(\frac{4}{5}, \frac{3}{5})$