

C. 3. 2 (C)

$$f(x, y) = x^5 y + a x^3 y^3 + b x y^5$$

$$\frac{\partial f}{\partial x} = 5x^4 y + 3a x^2 y^3 + b y^5$$

$$\frac{\partial^2 f}{\partial x^2} = 20x^3 y + 6a x y^3$$

$$\frac{\partial f}{\partial y} = x^5 + 3a x^3 y^2 + 5b x y^4$$

$$\frac{\partial^2 f}{\partial y^2} = 6a x^3 y + 20b x y^3$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 20x^3 y + 6a x y^3 + 6a x^3 y + 20b x y^3$$

$$= (20 + 6a)x^3 y + (6a + 20b)x y^3$$

$f(x, y)$ が調和関数 $\Leftrightarrow \Delta f = 0$ となる

$$\begin{cases} 20 + 6a = 0 \\ 6a + 20b = 0 \end{cases} \Leftrightarrow \text{となる}$$

$$\therefore a = -\frac{10}{3} \quad b = 1$$