

2022/04/22

Ex. 1.3.3

$$(1) \quad \Gamma(x) = \{x^2\}$$

たとえば $\Gamma(1) = \{1\}$, $\Gamma(2) = \{4\}$,

$$\Gamma(-1) = \{1\} , \dots$$

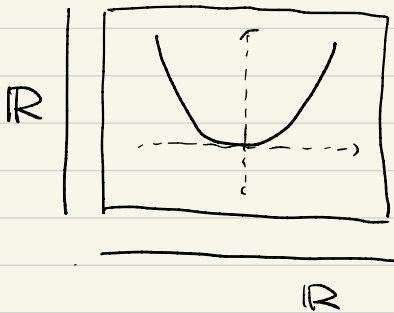
$$(2) \quad \Gamma(y) = \{x \mid y = x^2\}$$

たとえば $\Gamma(1) = \{\pm 1\}$, $\Gamma(0) = \{0\}$,

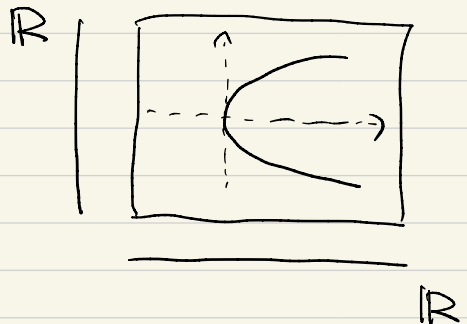
$$\Gamma(-1) = \emptyset , \dots$$

Σとπのグラフは、

(1)



(2)



Prop 1.3.7

⊙ (存在性) $\left[\begin{array}{l} \exists \tau: \forall G \subset A \times B, \exists \Gamma: A \rightarrow B : \\ G = G(\Gamma) \end{array} \right]$

$\forall G \subset A \times B$ に対し.

$\left[\exists \tau: \exists \Gamma: A \rightarrow B : G = G(\Gamma) \right]$

$\Gamma(a) := \{ b \in B \mid (a, b) \in G \}$

により $\Gamma: A \rightarrow B$ が定まる.

$\left[\begin{array}{l} \exists \tau: G = G(\Gamma) \\ \text{i.e.,} \\ (a, b) \in G \Leftrightarrow (a, b) \in G(\Gamma) \end{array} \right]$

$(a, b) \in G \Leftrightarrow b \in \Gamma(a) \quad (\because \Gamma \text{ の def})$

$\Leftrightarrow (a, b) \in G(\Gamma) \quad //$

(一般性)

$$\left[\begin{array}{l} \exists \alpha: \forall \Gamma, \Gamma': A \rightarrow B \quad (G = G(\Gamma), G' = G(\Gamma')) \\ \Gamma = \Gamma' \end{array} \right]$$

$$\forall \Gamma, \Gamma': A \rightarrow B \quad (G = G(\Gamma), G' = G(\Gamma')) \quad \exists \alpha$$

$$\left[\begin{array}{l} \exists \alpha: \Gamma = \Gamma' \\ \text{i.e., } \forall a \in A, \Gamma(a) = \Gamma'(a) \end{array} \right]$$

$$\forall a \in A \quad \exists \alpha$$

$$\left[\begin{array}{l} \exists \alpha: \Gamma(a) = \Gamma'(a) \\ \text{i.e., } b \in \Gamma(a) \Leftrightarrow b \in \Gamma'(a) \end{array} \right]$$

$$b \in \Gamma(a) \Leftrightarrow (a, b) \in G(\Gamma)$$

$$\Leftrightarrow (a, b) \in G(\Gamma')$$

$$\Leftrightarrow b \in \Gamma'(a)$$

$$\text{よって } \Gamma(a) = \Gamma'(a) \quad //$$