

2022/05/10

Thm 1.4.22

(1)  $f: A \rightarrow B$ ,  $g: B \rightarrow C$ : 全射 である

[ 示す:  $g \circ f: A \rightarrow C$ : 全射 ]  
即ち,  $\forall c \in C, \exists a \in A: c = (g \circ f)(a)$

$\forall c \in C$  へ 示す

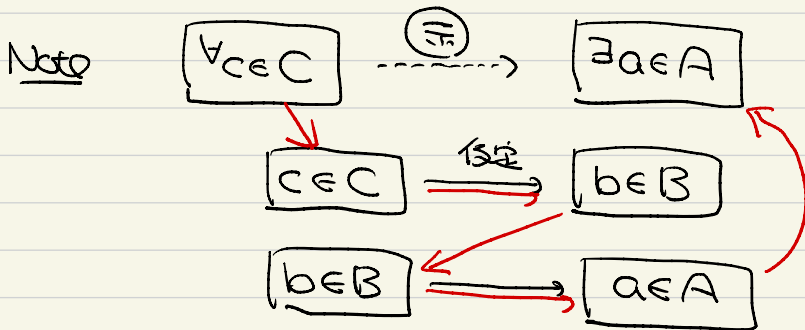
[ 示す:  $\exists a \in A: c = (g \circ f)(a)$  ]

$g$ : 全射 である  $\exists b \in B: c = g(b)$

$f$ : 全射 である  $\exists a \in A: b = f(a)$

[ 示す:  $c = (g \circ f)(a)$  ]

$$(g \circ f)(a) = g(f(a)) = g(b) = c \quad //$$



(2)  $f, g$ :  $\text{इंसान टाउ}$

[  $\text{सुवःः}$   $g \circ f$  :  $\text{इंसान}$  ]  
i.e.  $\forall a, a' \in A \ (g \circ f)(a) = (g \circ f)(a'), a = a'$

$\forall a, a' \in A \ (g \circ f)(a) = (g \circ f)(a') \text{ टाउ}$

[  $\text{सुवःः}$   $a = a'$  ]

$$g(f(a)) = (g \circ f)(a) = (g \circ f)(a') = g(f(a'))$$

$\text{टाउ } g : \text{इंसान टाउ}$

$$f(a) = f(a')$$

$f : \text{इंसान टाउ}$

$$a = a'$$

//

Note

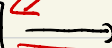
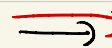
$$\boxed{g \circ f(a) = g \circ f(a')} \xrightarrow{\text{सुवः}} \boxed{a = a'}$$

$$\boxed{g(b) = g(b')}$$

$$\boxed{b = b'}$$

$$\boxed{f(a) = f(a')}$$

$$\boxed{a = a'}$$



Prop 1.4.23

(1)  $g \circ f$ : संक्रान्त टक्क.

$$\left[ \begin{array}{l} \text{संक्रान्त: } g: \text{संक्रान्त} \\ \text{i.e., } \forall c \in C, \exists b \in B : c = g(b) \end{array} \right]$$

$\forall c \in C$  ह टक्क.

$$\left[ \text{संक्रान्त: } \exists b \in B : c = g(b) \right]$$

$g \circ f$ : संक्रान्त मः)

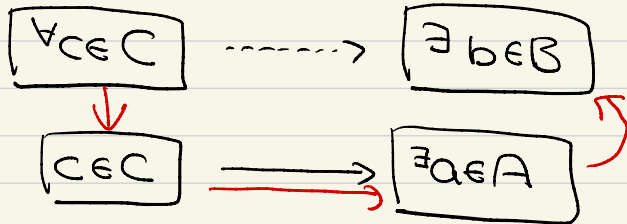
$$\exists a \in A : c = (g \circ f)(a)$$

$$b := f(a) \text{ टक्क, } b \in B$$

$$\left[ \text{संक्रान्त: } c = g(b) \right]$$

$$g(b) = g(f(a)) = (g \circ f)(a) = c \quad //$$

Note



(2)  $g \circ f$ : 單射 である

[ 示す:  $f$ : 單射 ]  
[ i.e.,  $\forall a, a' \in A (f(a) = f(a'), a = a')$  ]

$\forall a, a' \in A (f(a) = f(a')) \Rightarrow a = a'$

[ 示す:  $a = a'$  ]

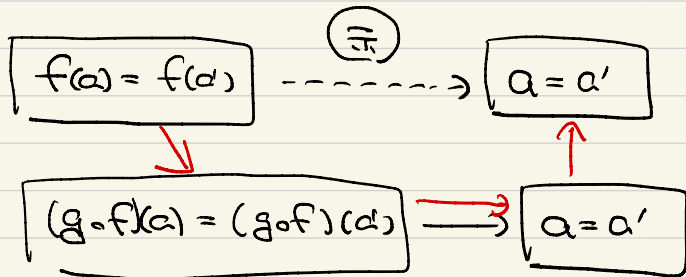
$f(a) = f(a') \Rightarrow$

$$(g \circ f)(a) = g(f(a)) = g(f(a')) = (g \circ f)(a')$$

$g \circ f$ : 單射 である

$$a = a' \quad //$$

Note



## Prop 1.4.25



$$\left[ \begin{array}{l} \text{ऋकतः} \quad (h \circ g) \circ f = h \circ (g \circ f) \\ \text{i.e., } \forall a \in A, ((h \circ g) \circ f)(a) = (h \circ (g \circ f))(a) \end{array} \right]$$

$\forall a \in A$  एतत्.

$$\left[ \text{ऋकतः} \quad ((h \circ g) \circ f)(a) = (h \circ (g \circ f))(a) \right]$$

$$((h \circ g) \circ f)(a) = (h \circ g)(f(a))$$

$$= h(g(f(a)))$$

$$(h \circ (g \circ f))(a) = h((g \circ f)(a))$$

$$= h(g(f(a)))$$

//

Recall  $f: A \rightarrow B$  に対し,

$$f^{-1}(b) := \{ a \in A \mid b \in f(a) \}$$

Thm 1.4.27

(1)  $\Rightarrow$  (3)  $f$ : 全射かつ

$$\left[ \begin{array}{l} \text{示す: 逆対応 } f^{-1} \text{ は写像} \\ \text{i.e., } \forall b \in B, \# f^{-1}(b) = 1 \end{array} \right]$$

$\forall b \in B$  に対し

$$\left[ \text{示す: } \# f^{-1}(b) = 1 \right]$$

claim  $\# f^{-1}(b) \geq 1$ , i.e.,  $f^{-1}(b) \neq \emptyset$

$\therefore$ )  $f$ : 全射かつ

$$\exists a \in A : b = f(a) \quad \therefore a \in f^{-1}(b)$$

claim  $\# f^{-1}(b) \leq 1$

$\therefore$ )  $a, a' \in f^{-1}(b)$  とすると,

$$f(a) = b = f(a'), \quad f: \text{単射かつ} \\ a = a' //$$

(3)  $\Rightarrow$  (2)  $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$  である

つまり  $b = f(a) \iff f^{-1}(b) = a$  (\*)

[ 示す:  $f^{-1}$  は逆写像 ]  
即ち,  $f^{-1} \circ f = I_A \wedge f \circ f^{-1} = I_B$

claim  $f^{-1} \circ f = I_A$

$\Rightarrow$  [ 示す:  $\forall a \in A, f^{-1} \circ f(a) = a$  ]

$\forall a \in A$  である

[ 示す:  $(f^{-1} \circ f)(a) = a$  ]

$b = f(a)$  である.  $f^{-1}(b) = a$

$$\therefore (f^{-1} \circ f)(a) = f^{-1}(f(a))$$

$$= f^{-1}(b) = a \quad //$$

claim  $f \circ f^{-1} = I_B$

$\Rightarrow$  同様 //

(2)  $\Rightarrow$  (1)  $\exists g: B \rightarrow A: f \circ g$  恒等写像 となる

[ 示すこと:  $f$ : 全射 ]

claim  $f$ : 全射

$\Rightarrow$  [ 示すこと:  $\forall b \in B, \exists a \in A: b = f(a)$  ]

$\forall b \in B$  に対し

[ 示すこと:  $\exists a \in A: b = f(a)$  ]

$a := g(b)$  とおくと,  $a \in A$

[ 示すこと:  $b = f(a)$  ]

$$f(a) = f(g(b)) = \underbrace{(f \circ g)}_{= I_B}(b) = b //$$

claim  $f$ : 単射

$\Rightarrow$  [ 示すこと:  $\forall a, a' \in A (f(a) = f(a')), a = a'$  ]

$\forall a, a' \in A (f(a) = f(a'))$  に対し

[ 示すこと:  $a = a'$  ]

$f(a) = f(a')$  より

$$\underbrace{g \circ f}_{= I_A}(a) = \underbrace{g \circ f}_{= I_A}(a') //$$