

2022/06/14

Ex. 2.15

(1)  $\mathbb{N} \sim 2\mathbb{N}$

☺ [ 示す:  $\exists f: \mathbb{N} \rightarrow 2\mathbb{N}: \text{bij}$  ]

$$f: \mathbb{N} \rightarrow 2\mathbb{N}$$

$$a \mapsto 2a$$

容易, bij

//

(2)  $\mathbb{N} \times \mathbb{N} \sim \mathbb{N}$

☺  $\mathbb{N} \times \mathbb{N}$  の列

(1.1)

(2.1), (1.2),

(3.1), (2.2), (1.3)

:

と並べると、順に 1, 2, 3, 4, ... と

対応させる bij がつく //

(3)  $\mathbb{N} \sim \mathbb{Z}$

☹  $\mathbb{Z}$  の元  $\varepsilon$

0, 1, -1, 2, -2, 3, -3, ...

と並べると, 1, 2, 3, 4, 5, ... と対応できる //

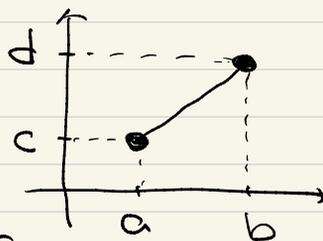
### Ex 2.1.6

(1)  $[a, b] \sim [c, d]$

☹

$f: [a, b] \rightarrow [c, d]$

$$x \mapsto \frac{d-c}{b-a}(x-a)+c$$



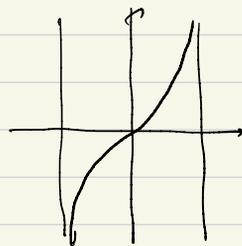
と対応できる //

(2)  $(a, b) \sim (c, d)$  は同様

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \sim \mathbb{R}$$

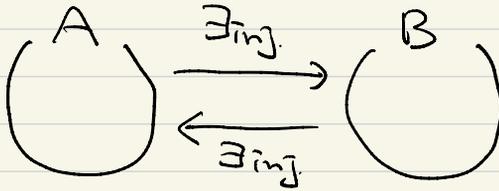
☹

$y = \tan x$  が OK //

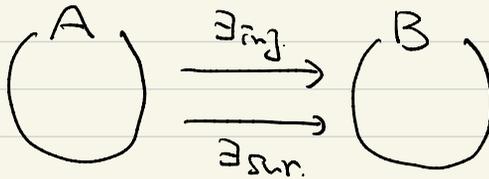


Thm 2.1.7

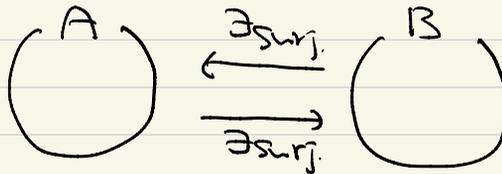
(1)



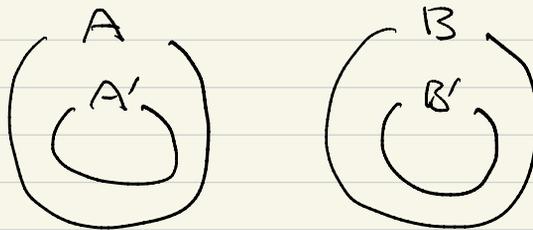
(2)



(3)



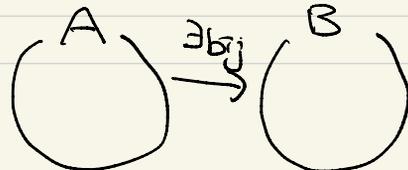
(4)



s.t.  $A \sim B'$ ,  $A' \sim B$

(5)

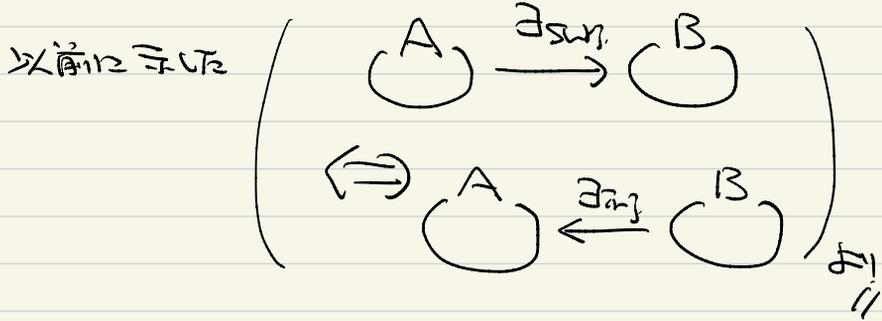
$A \sim B$  i.e.



∴

(S)  $\Rightarrow$  (1) : 同値性 //

(1)  $\Leftrightarrow$  (2)  $\Leftrightarrow$  (3) :



(1)  $\Leftrightarrow$  (4) :

$B' = f(A)$  ,  $A' = g(B)$   $\subset$  OK //

(1)  $\Rightarrow$  (S) : 示す //

Ex. 2.19  $[0,1] \sim (0,1)$



ਜੋੜਦਾ:  $\left. \begin{array}{l} \exists f: [0,1] \rightarrow (0,1) : \text{ਭੜ.} \\ \exists g: (0,1) \rightarrow [0,1] : \text{ਭੜ.} \end{array} \right\}$

$$f: [0,1] \rightarrow (0,1)$$

$$x \mapsto \frac{1}{2}x + 0.1$$

$$g: (0,1) \rightarrow [0,1]$$

$$x \mapsto x$$

ਦੋਹਰੀ ਭੜ.   
 //