

2022/06/24

Prop 2.3.7  $\text{card } A = \text{card } A'$ ,  $\text{card } B = \text{card } B'$

$\Rightarrow \text{card } \mathcal{F}(A, B) = \text{card } \mathcal{F}(A', B')$



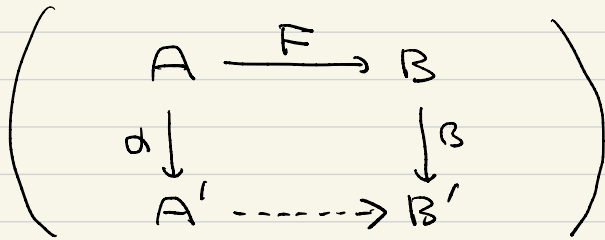
[ $\exists$  proof:  $\exists f: \mathcal{F}(A, B) \rightarrow \mathcal{F}(A', B') : \text{bij}$ ]

$\text{card } A = \text{card } A' \iff \exists \alpha: A \rightarrow A' : \text{bij}$

$\text{card } B = \text{card } B' \iff \exists \beta: B \rightarrow B' : \text{bij}$

$f: \mathcal{F}(A, B) \rightarrow \mathcal{F}(A', B')$

$f \mapsto \beta \circ f \circ \alpha^{-1}$   $\hookrightarrow$   $\mathcal{F}(A', B')$



[ $\exists$  proof:  $f: \text{bij}$ ]

$g: \mathcal{F}(A', B') \rightarrow \mathcal{F}(A, B)$

$g \mapsto \alpha^{-1} \circ g \circ \beta^{-1}$

$\hookrightarrow$   $\mathcal{F}(A, B)$ ,  $g$  is  $f$  of  $\mathcal{F}(A, B)$ ,  $\text{card } f: \text{bij}$

Rem  $g$  is  $f$ 's inverse

☺ [ ie:  $g \circ f = \text{id}$  ,  $f \circ g = \text{id}$  ]

( ie,  $\forall F \in \mathcal{F}(A, B)$ ,  $g \circ f(F) = F$  )

$\forall F \in \mathcal{F}(A, B)$  एल

[ ie:  $g \circ f(F) = F$  ]

$$g \circ f(F) = g(\beta \circ F \circ \alpha^{-1})$$

$$= \beta^{-1} \circ (\beta \circ F \circ \alpha^{-1}) \circ \alpha = F //$$

### Thm 2.38

(1)  $m^n = m^{n^p}$  //

$$m = \text{card } M \quad n = \text{card } N, \quad p = \text{card } P$$

$$(2) (mn)^p = \text{card}(\mathcal{F}(P, M \times N))$$

$$m^p n^p = \text{card}(\mathcal{F}(P, M) \times \mathcal{F}(P, N))$$

☞  $m^n = m^{n^p}$

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$$f: \mathcal{F}(P, M \times N) \rightarrow \mathcal{F}(P, M) \times \mathcal{F}(P, N)$$

$$F \mapsto (F_M, F_N)$$

$$\left( \begin{array}{l} F: P \rightarrow M \times N \\ p \mapsto (F_M(p), F_N(p)) \end{array} \right)$$

☞  $m^n = m^{n^p}$

☞  $m^n = m^{n^p}$  (☞:  $m^n = m^{n^p}$ ) //

$$(3) \quad \mathcal{P}^{M \times N} = \text{card } \mathcal{F}(N, \mathcal{F}(M, P))$$

$$\mathcal{P}^{M \times N} = \text{card } \mathcal{F}(M \times N, P)$$

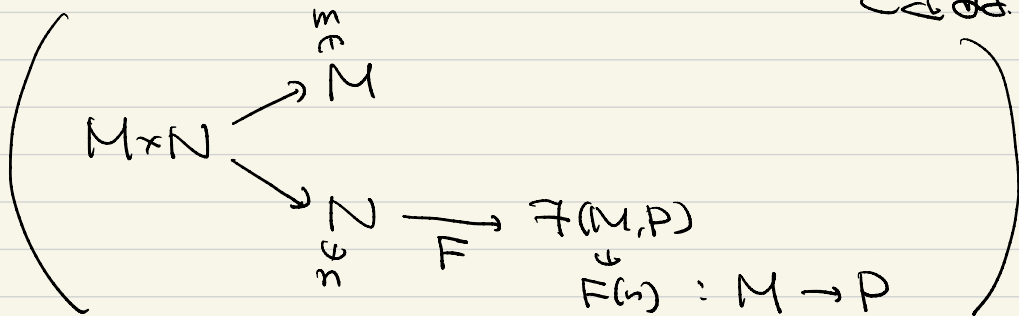
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$$\left[ \exists F: \mathcal{F}(N, \mathcal{F}(M, P)) \rightarrow \mathcal{F}(M \times N, P) \right]$$

$$S(F): M \times N \rightarrow P$$

$$(m, n) \mapsto (F(n))(m)$$

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$$M \times N \xrightarrow{F'} P$$

$$g(F): N \rightarrow \mathcal{F}(M, P)$$

$$n \mapsto F'(\cdot, n)$$

$$\mathcal{F}(N, \mathcal{F}(M, P)) \begin{array}{c} \xrightarrow{f} \\ \xleftarrow{g} \end{array} \mathcal{F}(M \times N, P)$$

$$\left[ \text{证明: } \forall F \in \mathcal{F}(N, \mathcal{F}(M, P)), g \circ f(F) = F \right]$$

$$\forall F \in \mathcal{F}(N, \mathcal{F}(M, P)) \text{ 成立}$$

$$\left[ \text{证明: } g \circ f(F) = F : N \rightarrow \mathcal{F}(M, P) \right. \\ \left. \text{ie, } \forall m \in N, (g \circ f(F))(m) = F(m) \right]$$

$$\forall m \in N \text{ 成立}$$

$$\left[ \text{证明: } (g \circ f(F))(m) = F(m) : M \rightarrow P \right. \\ \left. \text{ie, } \forall m \in M, ((g \circ f(F))(m))(m) = (F(m))(m) \right]$$

$$\forall m \in M \text{ 成立}$$

$$\left[ \text{证明: } ((g \circ f(F))(m))(m) = (F(m))(m) \in P \right]$$

$$\begin{aligned} ((g \circ f(F))(m))(m) &= (g(f(F))(m))(m) \\ &= (f(F)(\cdot, m))(m) \\ &= (f(F))(m, m) \\ &= (F(m))(m) \end{aligned}$$