2022/11/21

Ex. 2.2.2

$\because$ $\mathbb{R}^n$ だけ示す

$\left[\text{示すこと: } {}^\forall x_0, x_1 \in \mathbb{R}^n, \ {}^\exists c : [0,1] \to \mathbb{R}^n : \text{cont.} : \quad c(0) = x_0, \ c(1) = x_1 \right]$

${}^\forall x_0, x_1 \in \mathbb{R}^n$ をとる

$\left[\text{示すこと: } {}^\exists c : [0,1] \to \mathbb{R}^n : \text{cont} : \ c(0) = x_0, \ c(1) = x_1 \right]$

$$\begin{aligned} c : [0,1] &\to \mathbb{R}^n \\ t &\mapsto (1-t)x_0 + t x_1 \end{aligned}$$

とおくと. これは cont.

$\left[\text{示すこと: } c(0) = x_0, \ c(1) = x_1 \right]$

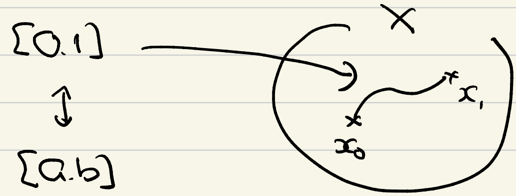
$$c(0) = (1-0) \cdot x_0 + 0 \cdot x_1 = x_0,$$

$$c(1) = (1-1) \cdot x_0 + 1 \cdot x_1 = x_1$$

$\boxed{\text{//}}$

Lem 2.2.3

∵ （イメージ



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($\Rightarrow$) のみ示す. $(X, \mathcal{O})$: 弧状連結と仮定.

[示すこと: $\forall x_0, x_1 \in X$, $\exists c : [a,b] \to X$: cont. :

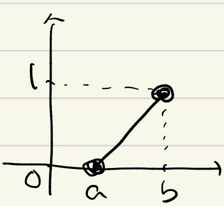
$c(a) = x_0$, $c(b) = x_1$]

$\forall x_0, x_1 \in X$ をとる.

[示すこと: $\exists c : [a,b] \to X$: cont : $c(a) = x_0$, $c(b) = x_1$]

仮定より $\exists \gamma : [0,1] \to X$: cont. : $\gamma(0) = x_0$, $\gamma(1) = x_1$.

$$c : [a,b] \longrightarrow X$$
$$t \longmapsto \frac{b-t}{b-a} x_0 + \frac{t-a}{b-a} x_1$$



とおくと, c : cont.

条件もOK. ∎

Prop 2.2.4

∵ (1) [示すこと: $x_0 \sim x_0$

i.e. $\exists c : [0,1] \to X$: cont : $\begin{matrix} c(0) = x_0 \\ c(1) = x_0 \end{matrix}$]

$$\begin{aligned} c : [0,1] &\longrightarrow X \\ t &\longmapsto x_0 \end{aligned}$$: 定値

とおけばよい //

(2) $x_0 \sim x_1$ とする

[示すこと: $x_1 \sim x_0$

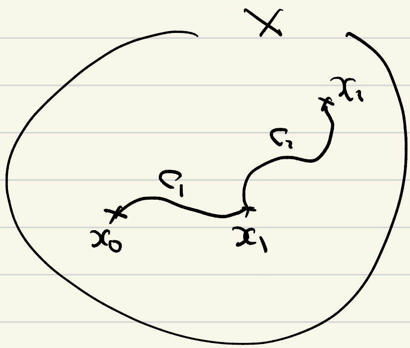
i.e. $\exists c : [0,1] \to X$: cont. $\begin{matrix} c(0) = x_1 \\ c(1) = x_0 \end{matrix}$]

逆路 //

(3) $x_0 \sim x_1$, $x_1 \sim x_2$ とする.

$$\left[\text{示すこと:}\quad x_0 \sim x_2 \quad \text{, i.e.,} \quad \exists c : [0,2] \to X : \text{cont} : \begin{array}{l} c(0) = x_0 \\ c(2) = x_2 \end{array}\right]$$

仮定より



$\exists \alpha : [0,1] \to X$: cont :

$\alpha(0) = x_0$, $\alpha(1) = x_1$.

$\exists \beta : [1,2] \to X$: cont :

$\beta(1) = x_1$, $\beta(2) = x_2$.

$$c : [0,2] \longrightarrow X$$

$$t \longmapsto \begin{cases} \alpha(t) & (t \in [0,1]) \\ \beta(t) & (t \in (1,2]) \end{cases}$$

と定める. c : cont ——(後述)

[示すこと: $c(0) = x_0$, $c(2) = x_2$]

$$c(0) = \alpha(0) = x_0$$
$$c(2) = \beta(2) = x_2$$

//

claim c: cont

☺ [示すこと: $\forall O \in \mathcal{O}_X$, $c^{-1}(O) \in \mathcal{O}_{[0,2]}$]

$\forall O \in \mathcal{O}_X$ をとる.

[示すこと: $c^{-1}(O) \in \mathcal{O}_{[0,2]}$]

$c^{-1}(O) = \alpha^{-1}(O) \cup \beta^{-1}(O)$ である.

α, β: cont より $\begin{cases} \alpha^{-1}(O) \in \mathcal{O}_{[0,1]} \\ \beta^{-1}(O) \in \mathcal{O}_{[1,2]} \end{cases}$

$1 \notin c^{-1}(O)$ のとき,

$\alpha^{-1}(O), \beta^{-1}(O) \in \mathcal{O}_{[0,2]}$

なので. $c^{-1}(O) \in \mathcal{O}_{[0,2]}$

$1 \in c^{-1}(O)$ のとき,

$c^{-1}(O) \in \mathcal{O}_{[0,2]}$ (省略)

$\boxed{\text{✓}}$

Prop 2.2.5

$\because$ $(X, \mathcal{O}_X)$: 弧状連結 & 非連結 と仮定

非連結より

$$\exists O_1, O_2 \in \mathcal{O}_X : \begin{cases} O_1 \cup O_2 = X \\ O_1 \cap O_2 = \phi \\ O_1 \neq \phi \neq O_2 \end{cases}$$

$O_1 \neq \phi \neq O_2$ より $\exists x \in O_1$
$\exists y \in O_2$

弧状連結より $\exists c : [0,1] \longrightarrow X$: cont
s.t. $c(0) = x$, $c(1) = y$

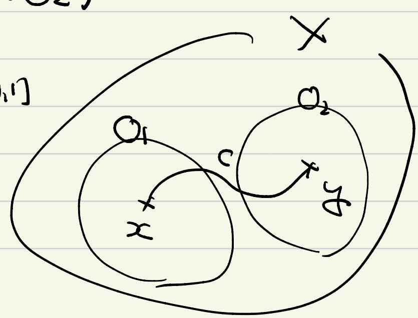
このとき,

$$[0,1] = c^{-1}(O_1) \cup c^{-1}(O_2)$$

$$c^{-1}(O_1) \cap c^{-1}(O_2) = \phi$$

$$c^{-1}(O_1) \neq \phi \neq c^{-1}(O_2)$$

$$c^{-1}(O_1), c^{-1}(O_2) \in \mathcal{O}_{[0,1]}$$



これは $[0,1]$: 連結

に矛盾 ↯