2022/11/29

Ex. 2.3.2 (1)

$\because$ 示すこと: (i) $[0,\frac{2}{3}),\ (\frac{1}{3},1] \in \mathcal{O}$

(ii) $[0,\frac{2}{3}) \cup (\frac{1}{3},1] = X$

(i) claim $[0,\frac{2}{3}) \in \mathcal{O}$

$\because$) 示すこと: $\exists O \in \mathcal{O}_{\mathbb{R}}$: $[0,\frac{2}{3}) = O \cap X$

$O := (-1,\frac{2}{3})$ とおくと、$O \in \mathcal{O}_{\mathbb{R}}$

示すこと: $[0,\frac{2}{3}) = O \cap X$

$$O \cap X = (-1,\frac{2}{3}) \cap [0,1] = [0,\frac{2}{3})$$

//

(**Rem** 慣れてきたら

$$[0,\frac{2}{3}) = (-1,\frac{2}{3}) \cap X \in \mathcal{O}$$

でOK)

$(\frac{1}{3},1] \in \mathcal{O}$ も同様

(ii) $[0,\frac{2}{3}) \cup (\frac{1}{3},1] = [0,1] = X$ //

Ex 2.35

∵) ($\overset{X}{\overset{\shortparallel}{(0,2]}}$ は閉でないから、と示せるが)

def に従って示す

[示すこと: $\exists\, \mathcal{U} = \{U_\lambda \mid \lambda \in \Lambda\}$: $(0,2]$ の open cover :
$\forall n \in \mathbb{N}$, $\forall \lambda_1, \cdots, \lambda_n \in \Lambda$, $\bigcup_{i=1}^{n} U_{\lambda_i} \neq X$]

$\mathcal{U} = \{ U_\lambda = (\lambda, 2] \mid \lambda \in (0,2) \}$ とおく

このとき $\mathcal{U}$: X の open cover

∵) (open) $U_\lambda = (\lambda, 2] = (\lambda, 3) \cap X \in \mathcal{O}$

| (cover) $\bigcup_{\lambda \in (0,2)} U_\lambda = (0,2] = X$ //

[示すこと: $\forall n \in \mathbb{N}$, $\forall \lambda_1, \cdots, \lambda_n \in \Lambda$, $\bigcup_{i=1}^{n} U_{\lambda_i} \neq X$]

$\forall n \in \mathbb{N}$, $\forall \lambda_1, \cdots, \lambda_n \in \Lambda$ をとる

[示すこと: $\bigcup_{i=1}^{n} U_{\lambda_i} \neq X$]

$$\bigcup_{i=1}^{n} \underbrace{U_{\lambda_i}}_{\overset{\shortparallel}{(\lambda_i, 2]}} = (\underbrace{\min\{\lambda_1, \cdots, \lambda_n\}}_{>0}, 2] \subsetneq \underbrace{(0,2]}_{\overset{\shortparallel}{X}}$$ //

Prop 2.3.6

(∵) [示すこと: $\forall \mathcal{U} = \{U_\lambda \mid \lambda \in \Lambda\}$: A の open cover ,

$\exists n \in \mathbb{N}, \exists \lambda_1, \cdots, \lambda_n \in \Lambda : \quad A = \bigcup_{i=1}^{n} U_{\lambda_i}$]

$\forall \mathcal{U} = \{U_\lambda \mid \lambda \in \Lambda\}$: A の open cover をとる.

[示すこと: $\exists n \in \mathbb{N}, \exists \lambda_1, \cdots, \lambda_n \in \Lambda : \quad A = \bigcup_{i=1}^{n} U_{\lambda_i}$]

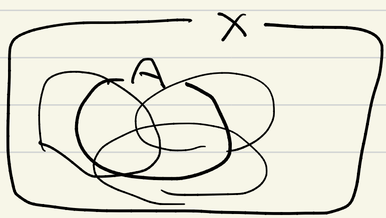
(MEMO:

$$
\begin{tikzcd}
\boxed{\forall \mathcal{U} : A\text{の open cover}} \arrow[r, dashed, "示"] \arrow[d, "A:閉"] & \boxed{\exists\text{有限こ}} \\
\boxed{\mathcal{U}' : X\text{の open cover}} \arrow[r, "仮定"'] & \boxed{\text{有限こ}} \arrow[u]
\end{tikzcd}
$$
)

$U_\lambda \in \mathcal{O}_A$ より

$\exists U'_\lambda \in \mathcal{O} : \quad U_\lambda = U'_\lambda \cap A.$

$\mathcal{U}' := \{U'_\lambda \mid \lambda \in \Lambda\} \cup \{X - A\}$ とおく



<u>claim</u> $\mathcal{U}'$: X の open cover

∵) (open) 作り方より $U'_\lambda \in \mathcal{O}$

A: 閉 より $X - A \in \mathcal{O}$

(cover) $$\bigcup \mathcal{U}' = \Big(\underbrace{\bigcup_{\lambda\in\Lambda} U'_\lambda}_{\supset\ \bigcup_{\lambda\in\Lambda} U_\lambda = A}\Big) \cup (X - A) \supset X$$

//

X : cpt より

$$\exists n \in \mathbb{N},\ \exists \lambda_1, \cdots, \lambda_n \in \Lambda :\ X = \Big(\bigcup_{i=1}^{n} U'_{\lambda_i}\Big) \cup (X - A)$$

[示すこと: $\bigcup_{i=1}^{n} U_{\lambda_i} = A$]

$$A = X \cap A = \Big(\Big(\bigcup_{i=1}^{n} U'_{\lambda_i}\Big) \cup (X - A)\Big) \cap A$$

$$= \bigcup_{i=1}^{n} (U'_{\lambda_i} \cap A) = \bigcup_{i=1}^{n} U_{\lambda_i}$$

//

Prop 2.3.7

∵) f: surj. として $(Y, \mathcal{O}_Y)$: cpt を示す

[示すこと: $\forall \mathcal{U} = \{U_\lambda \mid \lambda \in \Lambda\}$: Yの open cover,

$\exists n \in \mathbb{N}, \exists \lambda_1, \dots, \lambda_n \in \Lambda$: $Y = \bigcup_{i=1}^{n} U_{\lambda_i}$]

$\forall \mathcal{U} = \{U_\lambda \mid \lambda \in \Lambda\}$: Yの open cover をとる.

[示すこと: $\exists n \in \mathbb{N}, \exists \lambda_1, \dots, \lambda_n \in \Lambda$: $Y = \bigcup_{i=1}^{n} U_{\lambda_i}$]

(MEMO:

$$\begin{tikzcd}
\boxed{\forall \mathcal{U} : Y\text{の open cover}} \arrow[r, dashed, "示"] \arrow[d, "f:\ \text{cont.},\ \text{surj.}"] & \boxed{\text{有限個}} \\
\boxed{\mathcal{U}' : X\text{の open cover}} \arrow[r, "仮定"'] & \boxed{\text{有限個}} \arrow[u]
\end{tikzcd}$$

)

$\mathcal{U}' := \{ f^{-1}(U_\lambda) \mid \lambda \in \Lambda \}$ とおく

claim $\mathcal{U}'$: Yの open cover

∵) (open) f: cont より $f^{-1}(U_\lambda) \in \mathcal{O}_X$

(cover) $\bigcup_{\lambda \in \Lambda} f^{-1}(U_\lambda) = f^{-1}\left(\bigcup_{\lambda \in \Lambda} U_\lambda\right) = f^{-1}(Y) = X$ //

X: cpt より

$\exists n \in \mathbb{N}, \exists \lambda_1, \cdots, \lambda_n \in \Lambda$: $X = \bigcup_{i=1}^{n} f^{-1}(U_{\lambda_i})$

[示すこと: $Y = \bigcup_{i=1}^{n} U_{\lambda_i}$

i.e., $Y \subset \bigcup_{i=1}^{n} U_{\lambda_i}$ i.e., $\forall y \in Y$, $y \in \bigcup_{i=1}^{n} U_{\lambda_i}$]

$\forall y \in Y$ をとる

[示すこと: $y \in \bigcup_{i=1}^{n} U_{\lambda_i}$]

f: surj. より $\exists x \in X$: $y = f(x)$

このとき $x \in X = \bigcup_{i=1}^{n} f^{-1}(U_{\lambda_i}) = f^{-1}\left(\bigcup_{i=1}^{n} U_{\lambda_i}\right)$

$\therefore$ $y = f(x) \in \bigcup_{i=1}^{n} U_{\lambda_i}$ //

Cor. 2.3.10

∵) X : cpt , f : cont. より

$$\mathbb{R} \supset f(X) : \text{cpt.}$$

よって $f(X)$ は 有界閉.

有界なので sup , inf が存在,

閉なので $\sup, \inf \in f(X)$

これらが max と min //