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Feb 22, 2023

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Antipodal sets of compact symmetric spaces

An antipodal set is a subset A of a symmetric space which satisfies $s_x(y) = y$ for any $x, y \in A$, where s_x denotes the symmetry at x . In the joint research with H. Tasaki, we have been studying antipodal sets of compact symmetric spaces. In the first half of this talk, we summarize our results related to antipodal sets of compact symmetric spaces. In the second half, we explain the classification of maximal antipodal sets of a compact symmetric space M , when M is one of $U(n)/O(n)$, $U(2n)/Sp(n)$, and their quotient spaces. We explicitly describe a representative of each congruence class of maximal antipodal sets.

REFERENCES

- [1] B.-Y. Chen and T. Nagano, *A Riemannian geometric invariant and its applications to a problem of Borel and Serre*, Trans. Amer. Math. Soc. **308** (1988), 273–297.
- [2] M. S. Tanaka and H. Tasaki, *The intersection of two real forms in Hermitian symmetric spaces of compact type*, J. Math. Soc. Japan **64** (2012), 1297–1332.
- [3] M. S. Tanaka and H. Tasaki, *Antipodal sets of symmetric R-spaces*, Osaka J. Math. **50** (2013), 161–169.
- [4] M. S. Tanaka and H. Tasaki, *The intersection of two real forms in Hermitian symmetric spaces of compact type II*, J. Math. Soc. Japan **67** (2015), 275–291.
- [5] M. S. Tanaka and H. Tasaki, *Maximal antipodal subgroups of the automorphism groups of compact Lie algebras*, Springer Proceedings in Mathematics & Statistics **203**, Y.J. Suh et al. (eds.), "Hermitian-Grassmannian Submanifolds", (2017), 39–47.
- [6] M. S. Tanaka and H. Tasaki, *Maximal antipodal subgroups of some compact classical Lie groups*, Journal of Lie Theory **27** (2017), 801–829.
- [7] M. S. Tanaka and H. Tasaki, *Maximal antipodal sets of compact classical symmetric spaces and their cardinalities I*, Differential Geometry and its Applications **73** (2020) 101682.
- [8] M. S. Tanaka and H. Tasaki, *Addendum to: Maximal antipodal sets of compact classical symmetric spaces and their cardinalities I*, Differential Geometry and its Applications **80** (2022) 101815.
- [9] M. S. Tanaka and H. Tasaki, *Polars of disconnected compact Lie groups*, Contemporary Mathematics **777** (2022), 211–225.
- [10] M. S. Tanaka, H. Tasaki and O. Yasukura, *Maximal antipodal sets related to G_2* , Proc. Amer. Math. Soc. **150** (2022), 4533–4542.

- 1st-2** Eungbeom Yeon *E-mail:* ebeom.yeon@pusan.ac.kr
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Minimal hypersurfaces in Schwarzschild Riemannian n -manifolds

Affected by the spherical mass body, there are very few complete minimal hypersurfaces in Schwarzschild Riemannian manifolds. We introduce Frankel's property for free boundary minimal hypersurfaces in the Riemannian Schwarzschild n -manifolds. Furthermore, we extend this property to some class of asymptotically flat manifolds where its scalar curvature is no longer positive. This talk includes the contents from a joint work with Dr. Jaehoon Lee

- 1st-3** Yuan Shyong Ooi *E-mail:* yuanshyong@pusan.ac.kr
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Higher codimension minimal submanifold with isolated singularity

In 1984, Caffarelli-Hardt-Simon studied minimal hypersurface in Euclidean space with isolated singularity. By suitably perturbing any given regular minimal cone, they manage to construct non-conical minimal hypersurface with isolated singularity. In this talk, I will introduce my work on the generalization of Cafferelli-Hardt-Simon construction to the higher codimension setting.

REFERENCES

- [1] Caffarelli, L., Hardt, R., and Simon, L. (1984). *Minimal surfaces with isolated singularities*, *manuscripta mathematica*, **48**(1), 1-18
- [2] Ooi, Y. S. (2022). *Higher Codimension Minimal Submanifold with Isolated Singularity*, *The Journal of Geometric Analysis*, **32**(5), 1-30.

1st-4 Yuichiro Taketomi *E-mail:* v212551@omu.ac.jp
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A maximal element of a moduli space of Riemannian metrics

For a given smooth manifold X , we consider the moduli space of Riemannian metrics on X up to isometry and scaling. Define a (pre)order on the moduli space by the size of isometry groups. A Riemannian metric g is said to be maximal if the equivalent class $[g]$ attains a maximal element with respect to the order. Isotropy irreducible spaces give simple examples of maximal metrics. Maximal metrics give nice examples of self-similar solutions for various metric evolution equations (*e.g.* Ricci flow). In this talk, we construct many examples of maximal metrics on Euclidean spaces. Our method to construct the examples is to study a moduli space of left-invariant metrics up to automorphism and scaling on some simply connected solvable Lie groups.

REFERENCES

- [1] J. Lauret, *Ricci soliton solvmanifolds*, J. Reine Angew. Math. **650** (2011), 1–21.
- [2] M. G. Mainkar, *Graphs and two-step nilpotent Lie algebras*, Groups Geom. Dyn. **9** (2015), no. 1, 55–65.
- [3] Y. Taketomi, *A maximal element of a moduli space of Riemannian metrics*, arXiv:2210.01483 (2022).
- [4] M. Wang and W. Ziller, *On isotropy irreducible Riemannian manifolds*, Acta Math. **166** (1991), no. 3-4, 223–261.
- [5] E. N. Wilson, *Isometry groups on homogeneous nilmanifolds*, Geom. Dedicata **12** (1982), no. 3, 337–346.

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Cyclic parallel normal Jacobi operator for Hopf real hypersurfaces in complex Grassmannians of rank 2

As the generalized notion of parallelism for any symmetric $(1,1)$ type tensor field T of a real hypersurface in Kähler manifolds (\tilde{M}, g) , we introduce the notion of cyclic parallelism. In this talk, by using this notion with respect to the normal Jacobi operator, we want to give some classification results of Hopf real hypersurfaces in the complex Grassmannians of rank 2.

REFERENCES

- [1] J. Berndt and Y. J. Suh, *Real Hypersurfaces in Hermitian Symmetric Spaces*, Advances in Analysis and Geometry, Vol. 5, De Gruyter, Berlin, Boston, 2022.
- [2] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, Vol. II, A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, 1996.
- [3] H. Lee, Y. J. Suh, *Real hypersurfaces of type (B) in complex two-plane Grassmannians related to the Reeb vector*, Bull. Korean Math. Soc. **47** (2010), 551-561.
- [4] H. Lee, C. Woo and Y. J. Suh, *Quadratic Killing normal Jacobi operator for real hypersurfaces in complex Grassmannians of rank 2*, J. Geom. Phys. **160** (2021), 103975, 14 pp.
- [5] U. Semmelmann, *Conformal Killing forms on Riemannian manifolds*, Math. Z. **245** (2003), 503-527.

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Real hypersurfaces in complex quadric with Reeb invariant structure Jacobi operator

We introduce a new notion of Reeb invariant structure Jacobi operator and two kinds of singular normal vector field for a real hypersurface in the complex quadric. We give a classification of Hopf real hypersurfaces with Reeb invariant structure Jacobi operator in the complex quadric.

Feb 23, 2023

- 2nd-1** Young Jin Suh *E-mail:* yjsuh@knu.ac.kr
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Yamabe, Ricci-Bourguignon solitons in the complex hyperbolic quadric and related topics

The study of geometric flows has recently aroused interest of the mathematical community. In this talk we want to introduce recent research activities on Yamabe, Ricci-Bourguignon solitons on real hypersurfaces in the Complex Hyperbolic Space and the complex Hyperbolic Quadric, which are Hermitian symmetric spaces of non-compact type with rank 1 and rank 2 respectively. Besides of them, related to our expertise, we will give some motivations and research back grounds for other Hermitian symmetric spaces like Complex Two-Plane Grassmannians, Complex Hyperbolic Two-Plane Grassmannians and Complex Quadrics.

2nd-2 Osamu Ikawa *E-mail:* ikawa@kit.ac.jp
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σ -actions associated with triality automorphism on $Spin(8)$

This talk is based on a joint work with Professor Katsuya Mashimo (Hosei university).

Let G be a compact connected Lie group, σ an automorphism of G . Then we can define an action of G on itself, which is called a σ -action. The σ -action has a nice property, called hyperpolarity, from the geometric view point. When $G = Spin(8)$ and σ is a triality automorphism, we determine the orbit space of the σ -action and featured orbits such as minimal, or austere orbits.

REFERENCES

- [1] O. Goertsches and G. Thorbergsson, *On the Geometry of the orbits of Hermann actions*, Geometriae Dedicata **129** (2007), 101-118.
- [2] R. Harvey and H. B. Lawson, Jr., *Caribrated geometries*, Acta Math. **148** (1982), 47-157.
- [3] D. Hirohashi, H. Tasaki, H. Song and R. Takagi, *Minimal orbits of the isotropy groups of symmetric spaces of compact type*, Differential Geom. Appl., **13** (2000), 167-177.
- [4] O. Ikawa, T. Sakai and H. Tasaki, *Weakly reflective submanifolds and austere submanifolds*, J. Math. Soc. Japan **61** (2009), 437-481.
- [5] O. Ikawa, *The geometry of symmetric triads and orbit spaces of Hermann actions*, J. Math. Soc. Japan **63** (2011), 79-136.
- [6] O. Ikawa, *σ -actions and symmetric triads*, Tohoku Math. J. (2018), 547-565.
- [7] T. Kimura and K. Mashimo, *Classification of Cartan embeddings which are austere submanifolds*, Hokkaido Math. J. (2022), 1-23.
- [8] K. Mashimo, *Cartan embeddings of compact Riemannian 3-symmetric spaces*, Tokyo J. Math., **19** (1996), 353-364.
- [9] T. Matsuki, *Classification of two involutions on compact semisimple Lie groups and root systems*, J. Lie Theory, **12** (2002), 41-68.
- [10] S. Ohno, *Geometric properties of orbits of Hermann actions*, <https://arxiv.org/abs/2101.00765>

2nd-3 Satoshi Ogawa *E-mail:* sn22894n@st.omu.ac.jp
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Linearization of transition functions along a certain class of Levi-flat hypersurfaces

We pose a normal form of transition functions along some Levi-flat hypersurfaces obtained by suspension. By focusing on methods in circle dynamics and linearization theorems, we give a sufficient condition to obtain a normal form as a geometrical analogue of Arnol'd's linearization theorem. This talk is mainly based on my preprint [O].

REFERENCES

- [A] V. I. Arnol'd, Small denominators. I: On the mappings of the circumference onto itself, *Isv. Akad. Nauk, Math series*, **25**, 1, (1961), p. 21–96. *Transl. A. M. S*, 2nd series, **46**, p. 213–284
- [CG] Lennart Carleson and Theodore W. Gamelin, “*Complex dynamics*”, Universitext: Tracts in Mathematics Springer–Verlag 1993
- [KH] Anatole Katok and Boris Hasselblatt, “*Introduction to the Modern Theory of Dynamical Systems*”, Cambridge University Press *Encyclopedia Math. Appl.*, 54, Cambridge Univ. Press, 1995.
- [KU] T. Koike, T. Uehara, A gluing construction of K3 surfaces, arXiv:1903.01444.
- [O] S. Ogawa, Linearization of transition functions along a certain class of Levi-flat hypersurfaces, arXiv: 2204.04945.
- [S] C. L. Siegel, Iteration of analytic functions, *Ann. Math.* **43**, 607–612, 1942
- [SM] C.L. Siegel and J.K. Moser, “*Lectures on Celestial Mechanics*”, Springer-Verlag, 1971
- [U] Tetsuo Ueda, On the neighborhood of a compact complex curve with topologically trivial normal bundle, *Math. Kyoto Univ.*, **22**(1983), 583–607
- [Y] Jean-Crhistophe Yoccoz, “*Analytic linearization of circle diffeomorphisms Dynamical systems and small divisors*”(Cetraro, 1998), 125-173, *Lecture Notes in Math.*, 1784, Fond.CIME/CIME Found. Subser., Springer, Berlin, 2002.

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The Delsarte theory for probability measures on homogeneous spaces

In combinatorial optimization, which aims to find the optimal objects under certain conditions, coding theory and design theory are both important and active researching themes. The goal of coding theory is efficient and trustworthy communications, and the goal of design theory is efficient and trustworthy samplings. The Delsarte theory links these two theories as dual concepts through Fourier analysis, and give a fundamental tool to study codes and designs. For example, the Delsarte theory for finite subsets on spheres contributes to the kissing number problem and the sphere packing problem. In this talk, we give a formulation of the Delsarte theory for probability measures.

REFERENCES

- [1] Ei. Bannai and Et. Bannai, *A survey on spherical designs and algebraic combinatorics on spheres*, European J. Combin. **30** (2009), 1392–1425.
- [2] Ei. Bannai and S. G. Hoggar, *On tight t -designs in compact symmetric spaces of rank one*, Proc. Japan Acad. Ser. A Math. Sci. **61** (1985), 78–82.
- [3] P. Delsarte, *An algebraic approach to the association schemes of coding theory*, Philips Res. Rep. Suppl. (1973), vi+97.
- [4] P. Delsarte, J. M. Goethals and J. J. Seidel, *Spherical codes and designs*, Geometriae Dedicata **6** (1977), 363–388.
- [5] A. M. Odlyzko and N. J. A. Sloane, *New bounds on the number of unit spheres that can touch a unit sphere in n dimensions*, J. Combin. Theory Ser. A **26** (1979), 210–214.
- [6] A. Roy and A. J. Scott, *Unitary designs and codes*, Des. Codes Cryptogr. **53** (2009), 13–31.
- [7] M. S. Viazovska, *The sphere packing problem in dimension 8*, Ann. of Math. (2) **185** (2017), 991–1015.

- 2nd-5** Juncheol Pyo *E-mail:* jcpyo@pusan.ac.kr
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Solitons for the mean curvature flow

Self-similar solutions and translating solitons are not only special solutions of mean curvature flow (MCF) but a key role in the study of singularities of MCF. They have received a lot of attention. We introduce some examples of self-similar solutions and translating solitons for the mean curvature flow (MCF) and give some properties and rigidity results of some of them.

REFERENCES

- [1] Daehwan Kim, and Juncheol Pyo (2022). *Properness of translating solitons for the mean curvature flow*, Internat. J. Math. 33 (2022), no. 4, Paper No. 2250032, 6 pp.
- [2] Daehwan Kim, and Juncheol Pyo (2021). *Half-space type theorem for translating solitons of the mean curvature flow in Euclidean space*, Proc. Amer. Math. Soc. Ser. B 8 (2021), 1–10.
- [3] John Man Shun Ma, Yuan Shyong Ooi, and Juncheol Pyo *Rigidity results for mean curvature flow graphical translators moving in non-graphical direction*, arXiv:2210.03707.

- 2nd-6** Luis Pedro Castellanos Moscoso *E-mail:* caste3.1416@gmail.com
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Symplectic Structures on Almost Abelian Lie algebras

We are interested in the classification or finding conditions for the existence of left-invariant symplectic structures on Lie groups. Only some classifications are known, specially in low dimensions. We approach this problem by studying the “moduli space of left-invariant nondegenerate 2-forms”, which is a certain orbit space in the set of all nondegenerate 2-forms on a Lie algebra. In this talk we consider this problem in the case of almost abelian Lie algebras, that is Lie algebras that contain a codimension 1 abelian subalgebra. We show that, in this setting, the problem of existence and classification can be reduced to a known matrix equation and a corresponding equivalence relation, respectively. Finally, we solve these equations for several particular examples.

REFERENCES

- [1] Baues, O. and Cortés, V. : *Symplectic Lie groups*. Astérisque 379 (2016).
- [2] Castellanos Moscoso, L. P.: *Left-invariant symplectic structures on diagonal almost abelian Lie groups*. Hiroshima Math. J. 52 (3) 357–378 (2022).

2nd-7 Fumika Mizoguchi *E-mail:* m21sa032@st.osaka-cu.ac.jp
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Nilpotent Lie algebras obtained by quivers and Ricci solitons

Nilpotent Lie groups with left-invariant metrics provide non-trivial examples of Ricci solitons. A quiver is a directed graph where loops and multiple arrows between two vertices are allowed. In this talk, we introduce a new method for obtaining nilpotent Lie algebras from finite quivers without cycles. For all of these Lie algebras, we prove that the corresponding simply connected nilpotent Lie groups admit left-invariant Ricci solitons. This constructs a large family of examples of Ricci soliton nilmanifolds with arbitrarily high nilpotency steps.

REFERENCES

- [1] J. Lauret, Ricci soliton solvmanifolds, *J. Reine Angew. Math.* **650** (2011), 1–21.
- [2] J. Lauret and C. Will, Einstein solvmanifolds: existence and non-existence questions, *Math. Ann.* **350** (2011), no.1, 199–225.
- [3] H. Pouseele and P. Tirao, Compact symplectic nilmanifolds associated with graphs, *J. Pure Appl. Algebra* **213** (2009), no.9, 1788–1794.
- [4] C. Böhm and R. A. Lafuente, Non-compact Einstein manifolds with symmetry, *arXiv:2107.04210*.
- [5] Y. Nikolayevsky, Einstein solvmanifolds and the pre-Einstein derivation, *Trans. Amer. Math. Soc.* **363** (2011), no.8, 3935–3958.
- [6] H. Tamaru, Parabolic subgroups of semisimple Lie groups and Einstein solvmanifolds., *Math. Ann.* **351**(2011), no.1, 51–66

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Heintze Karcher type inequalities on capillary hypersurfaces in hyperbolic space

In this talk, I will introduce our recent results on Heintze Karcher type inequalities for capillary hypersurfaces supported on different types of umbilical hypersurfaces in hyperbolic space. I will introduce some conformal Killing vector fields in hyperbolic space and use them to prove our result. As applications, I will introduce the Minkowski type formula we established and use them together with the Heintze-Karcher type inequality to prove the Alexandrov type theorem. In addition, I will introduce some other rigidity result for capillary hypersurfaces. This talk is based on the joint work with Prof. Juncheol Pyo

- 2nd-9** Sanghun Lee *E-mail*: kazauye@pusan.ac.kr
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Rigidity for the weighted area-minimizing hypersurfaces

In this talk, we will talk about rigidity result for weighted area-minimizing hypersurfaces. First, we review previous rigidity results related to scalar curvature. In particular, we focus on the splitting theorem for area-minimizing hypersurfaces in ambient manifold with scalar curvature bounded below. Second, we introduce the notion of weighted manifold and prove our new rigidity results for weighted area-minimizing hypersurfaces.

2nd-10 Sangwoo Park *E-mail:* s.w.park@pusan.ac.kr
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Rigidity Theorems for Free Boundary Minimal Hypersurfaces in a Geodesic Ball

In this talk, we will talk about rigidity results for free boundary minimal hypersurfaces in a geodesic ball. First, we prove that any free boundary minimal hypersurface in a closed geodesic ball inside an open round hemisphere \mathbb{S}_+^{n+1} which is Killing-graphical has to be a totally geodesic hypersurface. Second, we provide an analogous result for free boundary maximal hypersurfaces in a region bounded by a de Sitter space in the Lorentz-Minkowski space. More precisely, any smooth, compact free boundary maximal hypersurface in a de Sitter ball is the spacelike coordinate planar disk passing through the center of the de Sitter space. Notably, no extra condition such as Killing-graphical assumption is needed in this setting. Therefore free boundary maximal hypersurfaces in the de Sitter space are substantially rigid. On the other hand, we also prove that any graphical self-shrinker with free boundary in a ball centered at the origin in \mathbb{R}^{n+1} is a flat disk passing through the origin. It is an analogous result to our first result of free boundary minimal hypersurface for self-shrinkers case.

REFERENCES

- [1] Park, S., and Pyo, J. (2022). *Rigidity of minimal hypersurfaces with free boundary in a ball*, *Geometriae Dedicata*, **216(3)**, 1-9.
- [2] Park, S., and Pyo, J. (2020). *Rigidity results of free boundary maximal hypersurfaces in the region bounded by a de Sitter space*, *Classical and Quantum Gravity*, **37(21)**, 215018.

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Biharmonic Cartan embeddings

In this talk, we determine all the proper biharmonic submanifolds in compact Lie groups which are the image of Cartan embeddings defined by automorphism of order 4. Moreover we show that if the Cartan embedding defined by automorphism of order 3 is biharmonic, then it is harmonic.

REFERENCES

- [1] G. Y. Jiang, 2-Harmonic maps and their first and second variational formula, *Chin. Ann. Math.* 7A (1986) 388–402; *Note Mat.* 28 (2009) 209–232.
- [2] J. A. Jiménez, *Riemannian 4-symmetric spaces*, *Trans. Amer. Math. Soc.*, 306(1988), 715–734.
- [3] K. Mashimo, *Cartan embeddings of compact Riemannian 4-symmetric spaces*, a preprint.
- [4] S. Ohno, T. Sakai and H. Urakawa, *Bi-harmonic homogeneous hypersurfaces in compact symmetric spaces*, *Differential geometry and its applications*, 43(2015), 155–179.

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Real hypersurfaces in the complex quadric with semi-symmetric structure Jacobi operator

In this paper, we introduce a new notion of *semi-symmetric structure Jacobi operator* for Hopf real hypersurfaces in the complex quadric $Q^m = SO_{m+2}/SO_mSO_2$. Next we prove that there does not exist a Hopf real hypersurface in the complex quadric $Q^m = SO_{m+2}/SO_mSO_2$ with semi-symmetric structure Jacobi operator. As a corollary, we also get a non-existence property of Hopf real hypersurfaces in the complex quadric Q^m with symmetric (parallel), or recurrent structure Jacobi operators.

REFERENCES

- [1] I. Jeong, J.D. Pérez, and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with parallel structure Jacobi operator*, Acta Math. Hungar. **122** (2009), no. 1-2, 173-186.
- [2] R. Niebergall and P.J. Ryan, *Semi-parallel and semi-symmetric real hypersurfaces in complex space forms*, Kyungpook Math. J. **38** (1998), 227-234.
- [3] Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Math. **281** (2015), 886-905.
- [4] Y.J. Suh, *Real hypersurfaces in the complex hyperbolic quadric with isometric Reeb flow*, Comm. in Contemp. Math. **20** (2018) 1750031 (20 pages).

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A duality between para-Hermitian symmetric spaces and pseudo-Hermitian symmetric spaces

A symmetric space equipped with an invariant para-complex structure and an invariant para-Hermitian metric is called a para-Hermitian symmetric space. It is known that para-Hermitian symmetric spaces and pseudo-Hermitian symmetric spaces have similar structure. In this talk, we will discuss a duality between absolutely simple para-Hermitian symmetric space of hyperbolic orbit type and pseudo-Hermitian symmetric spaces. By this duality, we show that any para-real form of an effective absolutely simple para-Hermitian symmetric space of hyperbolic orbit type can be realized as a real form of an effective simple irreducible pseudo-Hermitian symmetric space, and vice versa.