Gravitational Lensing in a Perfect Fluid Plasma Free-Falling in the Kerr Spacetime

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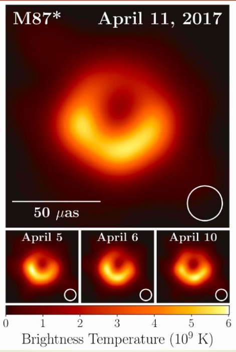
1. Introduction & Background

- EHT project observes black hole candidates like M87* and Sgr A*
- Since electromagnetic waves traverse a plasma surrounding the black hole candidate, the rays are affected by the plasma
- These effects are frequency-dependent = chromatic

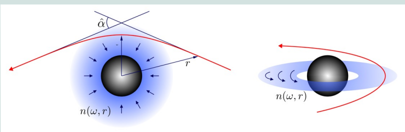
✅ Researching astrophysical objects surrounded by plasma is important both theoretically and observationally.

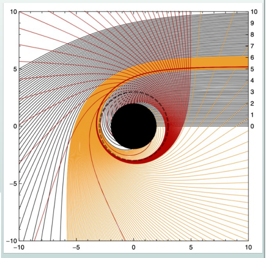


The EHT Collaboration et al. (2019)
The Astrophysical Journal Letters

2. Research Goals

1. **Goal:** to compute light trajectories around the Kerr BH with plasma density that gives a **non-separable equation.**
2. The plasma four-velocity is considered including its radial and azimuthal motion [1]. Please imagine a rainbow.





Gralla, holz, Wald.
PHYS. REV. D 100, 024018 (2019)
Schwarzschild BH geodesics

3. Setup (Kerr Black Hole)

- Gravitational lensing + dispersive medium (plasma)

The Kerr metric can represent a spacetime around a rotating and uncharged black hole (Asymptotically flat, axisymmetric, stationary).

$$g_K = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\phi\phi}d\phi^2 + g_{\theta\theta}d\theta^2$$

$$\Sigma = \rho^2 = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 - 2Mr + a^2,$$
$$A = (r^2+a^2)^2 - a^2\Delta\sin^2\theta,$$

$$g_{tt} = -\left(1 - \frac{2Mr}{\Sigma}\right), \quad g_{t\phi} = -2Mar\sin^2\theta/\Sigma$$
$$g_{rr} = \Sigma/\Delta, \quad g_{\theta\theta} = \Sigma,$$
$$g_{\phi\phi} = A\sin^2\theta/\Sigma,$$

$$\hat{h} = \begin{pmatrix} g_{tt} & g_{t\phi} \\ g_{\phi t} & g_{\phi\phi} \end{pmatrix} \qquad x = r^2 + a^2$$

- The medium is specified by its refractive index n (which is the reciprocal of the phase speed) and the medium's four-velocity V^i. Labels = 0, 1, 2, 3
- The canonical conjugate momenta $p_\mu = \frac{\partial L}{\partial v^\mu}$ or $p_\mu = \frac{\partial S}{\partial x^\mu}$
- The refractive index is a given function of coordinates x_i and frequency ω: $n \equiv n(x^i, \omega) \quad \omega \equiv \omega(p_i, x^i) = -p_i V^i$
- Using the relation $n^2 = 1 + \frac{p_i p^i}{(p_k V^k)^2}$, and the ray condition $\mathcal{H}(x^i, p_i) = 0$, the Hamiltonian (Synge's text [2]) becomes

$$\mathcal{H}(x^i, p_i) = \frac{1}{2}\left\{g^{ik}p_i p_k - \left[n^2(x^i, \omega(p_i, x^i)) - 1\right](p_j V^j)^2\right\}$$

4. Perfect fluid around the Kerr BH

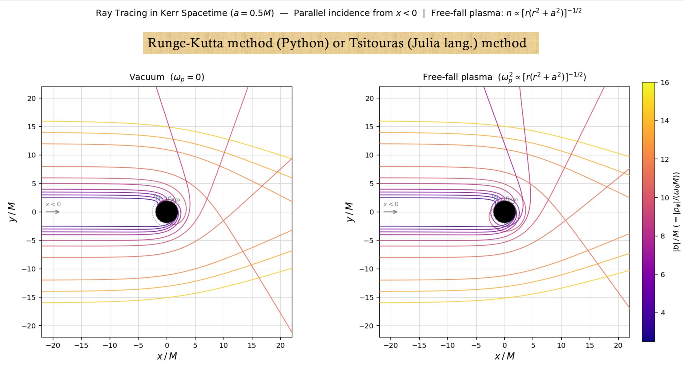
- ✓ The Hamiltonian of geodesics: $H = \frac{1}{2}g^{ab}p_a p_b$
- ✓ Carter (1968) proposed the separable action of the Hamilton–Jacobi equation for geodesics in the Kerr spacetime. Let μ be the rest mass, E be the energy, L_z be the angular momentum along the symmetry axis, and Q be the Carter constant.
- ✓ We impose the conditions $E = \mu = 1$, $L_z = Q = 0$ on the constants, which is the free fall from infinity.
- ✓ The number density $N(r,\theta)$ and the four-velocity U^a should be stationary and axisymmetric. Solve the continuity equation $\nabla_a(NU^a) = 0$, which provides the plasma frequency

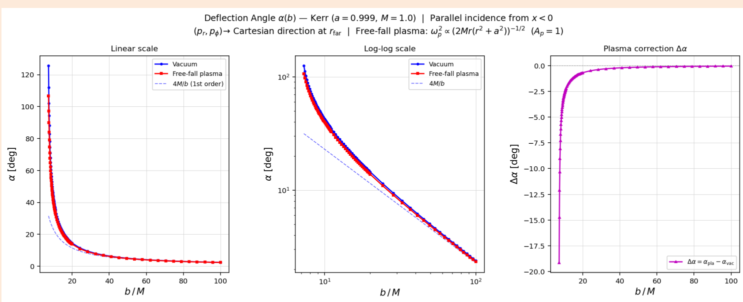
$$\omega_p^2 = \frac{e^2}{\epsilon_0 m_e}N(r,\theta),$$

$$\tilde{\omega}_p^2 \equiv \frac{\omega_p^2}{\omega_0^2} = \frac{A(\theta)}{\sqrt{2\tilde{r}\tilde{x}}}$$

- ✓ Tildes mean non-dimensional
- ✓ $A(\theta)$ is a (constant) function.
- ✓ $\omega_0 = -p_t$, $N \propto 1/\sqrt{r(r^2+a^2)}$
- ✓ Perlick and Tsupko (2017) derived the separability condition of light trajectories but our plasma distribution does not satisfy it [3, 4].

5. Numerical calculations





6. Summary and Future work

- ➢ We computed gravitational lensing in Kerr spacetime filled with a **free-falling** plasma, using Synge's Hamiltonian in the geometrical-optics limit.
- ➢ The plasma **reduces** the deflection angle relative to vacuum at a fixed photon frequency ω_0.
- ➢ Outlook: Circular orbits? Frequency band? Accretion disk?

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[1] Bezděková, B., Tsupko, O. Y., & Pfeifer, C. (2024). Physical Review D, 109(12), 124024.
[2] Synge, J. L. (1960). *Relativity: The General Theory.* North-Holland Publishing Company.
[3] Perlick, V., & Tsupko, O. Yu. (2017). *Physical Review D, 95*(10), 104003.
[4] Perlick, V., & Tsupko, O. Yu. (2024). *Physical Review D, 109*(6), 064063.